JOINT ESTIMATION OF STATE VARIABLES AND KINETICS PARAMETERS OF A DENITRIFICATION PROCESS

KARIM DAHECH, TARAK DAMAK AND AHMED TOUMI

Abstract This work was a contribution to the attempts aiming to solve the problems of estimation in biotechnological processes. These problems are related to the difficulty of on-line measurements of biological variables and to the fact that bioreactors are nonlinear time-varying processes with parameters uncertainties. For overcoming measurement difficulties, we proposed an adaptive estimation algorithm. This estimator is a combination of an asymptotic observer and an adaptive estimator. The objective of this estimator is to jointly estimate the states and the unknown kinetic parameters of a denitrification process. The process considered is a continuous-flow denitrification system in which a bacterial culture of Pseudomonas Denitrificans occurs: the biomass starts with consuming the acetic acid and the nitrate, and rejects some nitrites. Then, it continues to consume the acetic acid but the rejection of nitrites decreases. The simulation results seem to prove the performance and the robustness of the proposed estimation algorithm in spite of parameter variations and noisy measurements.

Key words, nonlinear system, adaptive estimator, asymptotic observer, Lyapunov function, denitrification process.

1. Introduction

During the control of biological processes several problems can be encountered. These problems are related to the difficulty of on-line measurements of biological variables, such as biomass concentration and specific biomass growth rate, on one hand, and to the fact that bioreactors are nonlinear time-varying processes with parameter uncertainties, on the other hand. Thus, with the purpose of process control, the knowledge of some parameters and state variables of the process is demanded. Therefore, the development of suitable algorithms to perform the estimation has captured the attention of many researchers. Several techniques have been introduced to estimate state variables and parameters from the available measurements, usually related to meaningful physico-chemical variables.

Although theories and applications for linear systems are well developed, the highly nonlinear nature of many chemical processes has given rise to nonlinear observers \[1, 2, 9, 12, 15, 16, 22, 23, 29\]. These observers are designed in such a way that they can cope with the intrinsic nonlinearities of the process dynamics. However, the construction of nonlinear observers still provides an open research field because the advance in the area of nonlinear observers often faces many typical obstacles such as very restrictive

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conditions to be satisfied, uncertainty in the performance and robustness and/or unsatisfactory estimates in the presence of noisy measurements.

The design and application of state observers in biological processes has been an active area over the past decades. In the literature, one can distinguish three classes of state estimators for biological processes: A first class of estimators whose development requires a perfect knowledge of the system parameters and in particular the kinetics of the process includes the extended Kalman filter and the extended Luenberger observers [5, 7, 9, 17, 18, 19, 24, 26]. The main drawback of these estimators consists in the difficulties to determine a priori its convergence and speed of convergence.

A second class of estimators whose development is based on the dynamic model of the process but without requiring the knowledge of the kinetics of the process consists of the asymptotic observer [3, 9-11]. The potential drawback of the asymptotic observer is the rate of convergence of the estimation which fully depends on the operating conditions. The methodology is based on the definition of a general dynamic model particularly adopted with continuous-flow biological processes. The suitable choice of a distribution of this model then makes it possible to create an observer independent of the knowledge of the kinetic variables.

A third class comprises the estimators allowing the joint estimation of the states and the parameters [4, 8, 13, 20]. For this type of estimator, the model of the kinetics of the process is known but the parameters of which it depends are unknown.

In this work, we developed an estimator of the third class for the estimation of the states and the kinetic parameters of a denitrification process. This estimator is a combination of an asymptotic observer and an adaptive estimator. The stability of the estimator is studied by using the method of Lyapunov. The next section will present the model of the denitrification process and the problem statement. Section 3, will show the development of the considered estimator. Section 4 will deal with the application of this estimator to the denitrification process. Simulation results are illustrated and discussed in order to prove the performances of this estimator. Finally, section 5 will end with conclusions.

2. Process model and problem statement
2.1. Process model

The considered process is a continuous-flow denitrification process in which a bacterial culture of *Pseudomonas Denitrificans* occurs: the biomass $X$ starts with consuming the acetic acid $S_3$ and the nitrate $S_1$ and rejects some nitrites $S_2$. Then, it continues to consume the acetic acid but the production of nitrites decreases. The mathematical dynamical model of the process is [12]:

$$
\begin{align*}
\dot{S}_1 &= -y_{11} \mu_1 X + D(S_{in} - S_1) \\
\dot{S}_2 &= (y_{12} \mu_1 - y_{22} \mu_2) X + D(S_{2in} - S_2) \\
\dot{S}_3 &= -(y_{13} \mu_1 + y_{23} \mu_2) X + D(S_{3in} - S_3) \\
\dot{X} &= (\mu_1 + \mu_2) X - k_d X - DX
\end{align*}
$$

where $S_1$, $S_2$, $S_3$ and $X$ are the concentrations of the corresponding species; $\mu_1$ and $\mu_2$ represent the specific growth rate of the biomass respectively on the acetic acid and the nitrite; $k_d$ is the coefficient of mortality, $S_{in}$, $S_{2in}$ and $S_{3in}$ are the input
concentrations respectively of $S_1$, $S_2$ and $S_3$; $D$ is the dilution rate and finally \{\gamma_{ij}\} are yield coefficients.

The expressions of the two specific growth rates $\mu_1$ and $\mu_2$ are given by:

$$\mu_1 = \mu_{1\text{max}} \frac{S_1}{(S_3 + k_{S_1})(S_1 + k_{S_1})},$$

$$\mu_2 = \mu_{2\text{max}} \frac{S_2}{(S_3 + k_{S_2})(S_2 + k_{S_2})}$$

where $\mu_{1\text{max}}$ and $\mu_{2\text{max}}$ are the maximal values of $\mu_1$ and $\mu_2$; $k_{S_i}$, $k_{\tilde{S}_i}$ and $k_{\tilde{S}_i}$ are the constants of affinity associated respectively to the nitrate, to the nitrite and to the acetic acid.

### 2.2. Problem statement

In this work, we present our contribution for the joint estimation of state variables and parameters of a denitrification process. We will develop an adaptive estimator based on the structure of the dynamic model of the considered process. This estimator is a combination of an asymptotic observer [9] and an adaptive estimator [8]. The development of this estimator is based on the following hypotheses:

- The yield coefficients \{\gamma_{ij}\} are known.
- The two specific growth rates $\mu_1$ and $\mu_2$ are two nonlinear functions depending on the concentrations $S_1$, $S_2$ and $S_3$ and two vectors of parameters $\theta_1$ and $\theta_2$, which are unknown; one has as follows: $\mu_1 = \mu_1(S_1, S_2, \theta_1)$ and $\mu_2 = \mu_2(S_2, S_3, \theta_2)$.
- The derivatives of $\mu_1$ with respect to $\theta_1$ and $S_3$ are bounded.
- The derivatives of $\mu_2$ with respect to $\theta_2$ and $S_3$ are bounded.
- We assume that the concentrations of nitrate and nitrite are measured components and the concentrations of the acetic acid and the biomass are unmeasured components.

### 3. Joint estimation of states and parameters

The model of state estimator is given by the following system:

$$\begin{aligned}
\dot{\hat{S}}_1 &= -y_{11}\hat{\mu}_1 \hat{X} + D(S_{1\text{in}} - \hat{S}_1) - \alpha_1\hat{S}_1 \\
\dot{\hat{S}}_2 &= (y_{12}\hat{\mu}_1 - y_{22}\hat{\mu}_2) \hat{X} + D(S_{2\text{in}} - \hat{S}_2) - \alpha_2\hat{S}_2 \\
\dot{\hat{S}}_3 &= -(y_{13}\hat{\mu}_1 + y_{23}\hat{\mu}_2) \hat{X} + D(S_{3\text{in}} - \hat{S}_3) - \alpha_3\hat{S}_1 - \alpha_4\hat{S}_2 \\
\dot{\hat{X}} &= (\hat{\mu}_1 + \hat{\mu}_2) \hat{X} - k_{\tilde{S}_1}\hat{X} - D\hat{X} - \alpha_5\hat{S}_1 - \alpha_6\hat{S}_2
\end{aligned}$$

where $\hat{S}_1$, $\hat{S}_2$, $\hat{S}_3$, $\hat{X}$, $\hat{\mu}_1$ and $\hat{\mu}_2$ are respectively the estimates of $S_1$, $S_2$, $S_3$, $X$, $\mu_1$ and $\mu_2$. Let us consider $\hat{S}_1$ and $\hat{S}_2$ the estimation errors respectively of $S_1$ and $S_2$: $\hat{S}_1 = S_1 - \hat{S}_1$ and $\hat{S}_2 = S_2 - \hat{S}_2$. \{\alpha_i\}_{i=1,6} are the estimated gains.
The structure of the specific growth rates appearing in the estimation model (3) is the same one as that of the original model (1).

We have

\[ \hat{\mu}_1 = \mu_1 \left( \hat{\theta}_1, S_1, \hat{S}_3 \right) \quad \text{and} \quad \hat{\mu}_2 = \mu_2 \left( \hat{\theta}_2, S_2, \hat{S}_3 \right) \]

(4)

Thus, the estimation problem is to determine the gains \( \alpha_i \) for \( i = 1 \ldots 6 \) in order to reduce the estimation error of the state variables and the specific growth rates towards zero. Without specifying the expression of \( \mu_1 \) and \( \mu_2 \), estimation error system can be obtained by using a Taylor series development to the order one of \( \mu_1 \) and \( \mu_2 \) [27, 28].

We can write:

\[ \hat{\mu}_1 - \mu_1 = H_1^T \hat{\theta}_1 + K_1 \hat{S}_3 \quad \text{and} \quad \hat{\mu}_2 - \mu_2 = H_2^T \hat{\theta}_2 + K_2 \hat{S}_3 \]

(5)

with:

\[ \hat{\theta}_1 = \hat{\theta}_1 - \theta_1 \in m_1 \quad \text{and} \quad \hat{\theta}_2 = \hat{\theta}_2 - \theta_2 \in m_2 \]

(6)

\[ H_1 = \frac{\partial \mu_1}{\partial \theta_1} \left\| \begin{array}{c} 4, S_1, S_3 \end{array} \right\| \quad \text{and} \quad H_2 = \frac{\partial \mu_2}{\partial \theta_2} \left\| \begin{array}{c} 2, S_2, S_3 \end{array} \right\| \]

(7)

\[ K_1 = \frac{\partial \mu_1}{\partial S_3} \left\| \begin{array}{c} 4, S_1, S_3 \end{array} \right\| \quad \text{and} \quad K_2 = \frac{\partial \mu_2}{\partial S_3} \left\| \begin{array}{c} 2, S_2, S_3 \end{array} \right\| \]

(8)

\( m_1 \) and \( m_2 \) are respectively the rows of the two vectors \( \theta_1 \) and \( \theta_2 \).

While deducting (1) from (3) and using (6), we obtain the following system error:

\[ \begin{cases} \dot{\hat{S}}_1 = -(D + \alpha_1) \hat{S}_1 - y_{11} \hat{X} \hat{K}_1 \hat{S}_3 - y_{11} \mu_1 \hat{X} - y_{11} \hat{X} H_1^T \hat{\theta}_1 \\ \dot{\hat{S}}_2 = -(D + \alpha_2) \hat{S}_2 + \left( y_{12} K_1 - y_{22} K_2 \right) \hat{X} \hat{S}_3 \\ \quad - \left( y_{12} \mu_1 - y_{22} \mu_2 \right) \hat{X} + y_{12} \hat{X} H_1^T \hat{\theta}_1 - y_{22} \hat{X} H_2^T \hat{\theta}_2 \\ \dot{\hat{S}}_3 = -\alpha_3 \hat{S}_1 - \alpha_4 \hat{S}_2 - \left( D + y_{13} \hat{X} \hat{K}_1 + y_{23} \hat{X} K_2 \right) \hat{S}_3 \\ \quad - \left( y_{13} \mu_1 + y_{23} \mu_2 \right) \hat{X} - y_{13} \hat{X} H_1^T \hat{\theta}_1 - y_{23} \hat{X} H_2^T \hat{\theta}_2 \\ \dot{\hat{X}} = -\alpha_5 \hat{S}_1 - \alpha_6 \hat{S}_2 + \left( K_1 + K_2 \right) \hat{X} \hat{S}_3 - \left( \mu_1 + \mu_2 - k_d - D \right) \hat{X} \\ \quad + \hat{X} H_1^T \hat{\theta}_1 + \hat{X} H_2^T \hat{\theta}_2 \end{cases} \]

(9)

The system error (9) can be written in the following matrix form:

\[ \dot{\epsilon} = \Omega \epsilon + \Lambda \Psi \tilde{\theta} \]

(10)

with:

\[ \epsilon = \begin{bmatrix} \hat{S}_1 & \hat{S}_2 & \hat{S}_3 & \hat{X} \end{bmatrix}^T \quad \text{and} \quad \tilde{\theta} = \begin{bmatrix} \hat{\theta}_1 & \hat{\theta}_2 \end{bmatrix}^T \]
\[
\Omega = \begin{bmatrix}
-(D + \alpha_1) & 0 & -y_{11}\dot{X}K_1 & -y_{11}\mu_1 \\
0 & -(D + \alpha_2) & (y_{12}K_1 - y_{22}K_2)\dot{X} & y_{12}\mu_1 - y_{22}\mu_2 \\
-\alpha_3 & -\alpha_4 & -(D + y_{13}\dot{X}K_1 + y_{23}\dot{X}K_2) & -(y_{13}\mu_1 + y_{23}\mu_2) \\
-\alpha_5 & -\alpha_6 & (K_1 + K_2)\dot{X} & \mu_1 + \mu_2 - k_d - D
\end{bmatrix}
\]

\[
\Lambda = \begin{bmatrix}
-y_{11} & 0 \\
y_{12} & -y_{22} \\
y_{13} & -y_{23} \\
1 & 1
\end{bmatrix}
\]

and

\[
\Psi = \begin{bmatrix}
\dot{X}H_1^T \\
\dot{X}H_2^T
\end{bmatrix}
\]

For the determination of the estimating gains \( \alpha_i \) for \( i = 1 \ldots 6 \), let us study the stability of the matrix \( \Omega \).

Let us suppose that the elements \( \Omega_{ij} \) and their derivatives \( \frac{d\Omega_{ij}}{dt} \) are bounded [25].

The characteristic polynomial of this matrix is written in the following form:

\[
P(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0
\]

(11)

The stability study of the matrix \( \Omega \) is difficult because the sign of the coefficients \( a_i \) for \( i = 0 \ldots 3 \) is unknown. For overcoming this problem, we will combine the estimation algorithm (3) with an asymptotic observer. This combination enables us to decrease the number of estimated gains and to simplify the matrix \( \Omega \).

In the following paragraph, we present the asymptotic observer.

### 3.1. Asymptotic observer

The general idea of this approach consists in representing the evolution of the process by the following general dynamic model [9]:

\[
\ddot{\xi} = K\Phi(\dot{\xi}) - D\ddot{\xi} - Q(\dot{\xi}) + F
\]

(12)

where \( \dot{\xi} \in \mathbb{R}^n \) is the vector of the concentration of the process components; \( K \in \mathbb{R}^{nxn} \) is the yield coefficients matrix; \( \Phi(\dot{\xi}) \) is the reaction rate vector; \( D \in \mathbb{R} \) is the dilution rate; \( F \in \mathbb{R}^n \) is the feed rate vector and \( Q \in \mathbb{R} \) is the gaseous outflow rate vector.

This observer, known as asymptotic, supposes that the structure of the reaction kinetics is unknown. For the design of this observer, the authors are assuming the following [4, 6]:

- The yield coefficients are known constants,
- The dimension of the measured state vector is higher than or equal to the row of the matrix \( K \).

The synthesis of the observer is based on the decomposition of the state vector \( \dot{\xi} \) in two vectors \( \dot{\xi}_m \) of the measured state variables and \( \dot{\xi}_u \) the unmeasured state variables. Thus, the above general dynamic model (12) can then be rewritten as follows [9]:
\[ \dot{\xi}_m = K_m \Phi(\xi) - D\dot{\xi}_m - Q_m(\xi) + F_m \]
\[ \dot{\xi}_nn = K_{nn} \Phi(\xi) - D\dot{\xi}_{nn} - Q_{nn}(\xi) + F_{nn} \]  \( \text{(13)} \)

where, \( Q = \begin{bmatrix} Q_m \\ Q_{nn} \end{bmatrix} \) and \( F = \begin{bmatrix} F_m \\ F_{nn} \end{bmatrix} \) respectively denote the partitions of \( K, Q \) and \( F \) induced by the partition of \( \xi \).

If the matrix \( K_m \) is full rank, then we can define the following linear state transformation:
\[ Z = A_0\xi_m + \xi_{nn} \]  \( \text{(14)} \)

Where the matrix \( A_0 \) is the single solution of the matrix equation:
\[ A_0K_m + K_{nn} = 0 \]  \( \text{(15)} \)

Thus, the algorithm of the asymptotic observer is represented by the following equations system:
\[ \begin{cases} \dot{\hat{Z}} = -D\dot{\hat{Z}} + A_0 \left( F_m - Q_m(\xi) \right) + \left( F_{nn} - Q_{nn}(\xi) \right) \\ \hat{\xi}_{nn} = \dot{\hat{Z}} - A_0\hat{\xi}_m \end{cases} \]  \( \text{(16)} \)

3.2. State variables and parameters estimation
3.2.1. State variables estimation

The model of the denitrification process (1) can be written in the form of the general dynamic model (12) with:
\[ K = \begin{bmatrix} -y_{11} & 0 \\ y_{12} & -y_{22} \\ -y_{13} & -y_{23} \\ 1 & 1 \end{bmatrix}; \quad \Phi(\xi) = \begin{bmatrix} \mu_1X \\ \mu_2X \end{bmatrix}; \quad Q = \begin{bmatrix} 0 & 0 & k_jX \end{bmatrix}^T \]
\[ F = \begin{bmatrix} DS_{1in} & DS_{2in} & DS_{3in} & 0 \end{bmatrix}^T \]

Let us consider \( \xi_m = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \) the vector of the measured state variables and \( \xi_{nn} = \begin{bmatrix} S_3 \\ X \end{bmatrix} \) the vector of the unmeasured state variables. The system (1) can be written in the form (13) with:
\[ K_m = \begin{bmatrix} -y_{11} & 0 \\ y_{12} & -y_{22} \end{bmatrix}, \quad K_{nn} = \begin{bmatrix} -y_{13} & -y_{23} \end{bmatrix}, \quad F_m = \begin{bmatrix} DS_{1in} \\ DS_{2in} \end{bmatrix}, \quad F_{nn} = \begin{bmatrix} DS_{3in} \\ 0 \end{bmatrix}, \]
\[ Q_m = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{and} \quad Q_{nn} = \begin{bmatrix} 0 \\ k_jX \end{bmatrix} \]

The matrix \( K_m \) is full rank. Therefore there is a linear state transformation defined by:
\[ Z = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = A_0 \xi_m + \xi_{nm} \]  
(17)

Where the matrix \( A_0 = -K_m^{-1} \) is given by the following expression:

\[ A_0 = \frac{1}{y_{11} y_{22}} \begin{bmatrix} -y_{13} y_{22} - y_{23} y_{12} & -y_{11} y_{23} \\ y_{22} + y_{12} & y_{11} \end{bmatrix} \]  
(18)

The algorithm of the asymptotic observer applied to the denitrification process is given by the following equations:

\[ \dot{\hat{Z}} = \begin{bmatrix} \dot{\hat{Z}}_1 \\ \dot{\hat{Z}}_2 \end{bmatrix} = -D \begin{bmatrix} \hat{Z}_1 \\ \hat{Z}_2 \end{bmatrix} + A_0 \begin{bmatrix} DS_{1m} \\ DS_{2m} \end{bmatrix} + \begin{bmatrix} 0 \\ k_a X \end{bmatrix} \]  
(19)

\[ \dot{\xi}_{m} = \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} - A_0 \begin{bmatrix} \hat{S}_1 \\ \hat{S}_2 \end{bmatrix} \]  
(20)

Now, let us take again the error system (9) while replacing \( \hat{S}_3 \) and \( \hat{X} \) by \( \dot{\hat{Z}}_1 \) and \( \dot{\hat{Z}}_2 \); \( \hat{S}_3 \) and \( \hat{X} \) by their expressions obtained from the equations (17) and (20). Thus, the error system is rewritten in the following form:

\[ \dot{Y} = A \tilde{Y} + B \]  
(21)

with:

\[ A = \begin{bmatrix} a_1 & a_2 & -y_{11} \hat{X} K_1 & -y_{11} \mu_1 \\ a_2 & a_5 & (y_{12} K_1 - y_{22} K_2) \hat{X} & y_{12} \mu_1 - y_{23} \mu_2 \\ 0 & 0 & -D & 0 \\ 0 & 0 & 0 & -D \end{bmatrix}, \quad B = \begin{bmatrix} -y_{11} H_1^T \hat{X} \hat{\theta}_1 \\ y_{12} H_1^T \hat{X} \hat{\theta}_1 - y_{22} H_2^T \hat{X} \hat{\theta}_2 \\ 0 \\ 0 \end{bmatrix} \]

\[ \dot{Y} = \begin{bmatrix} \dot{\hat{S}}_1 \\ \dot{\hat{S}}_2 \\ \dot{\hat{Z}}_1 \\ \dot{\hat{Z}}_2 \end{bmatrix}^T \tilde{Y} = \begin{bmatrix} \tilde{S}_1 \\ \tilde{S}_2 \\ \tilde{Z}_1 \\ \tilde{Z}_2 \end{bmatrix}^T \]

\[ a_1 = -D + (a_1) - (y_{11} K_1 + y_{23} K_2) \hat{X} \]

\[ a_2 = -(y_{12} \mu_1 + y_{23} \mu_2) \]

\[ a_5 = (K_1 + K_2) \hat{X} \]

\[ a_6 = -D + (a_2) + \mu_1 + \mu_2 \]

The characteristic polynomial of the matrix \( A \) is given by the following equation:

\[ P(\lambda) = (\lambda + D)^2 \left( \lambda^2 - (a_1 + a_6) \lambda + a_4 \right) \]  
(22)

According to (22), the system described by the matrix \( A \) is stable if and only if:
\[ \lambda^2 - (a_i + a_6)\lambda + a_i a_6 - a_2 a_5 = 0 \]

has two roots located in the left-half of the complex plane.

Let us suppose that the elements \( A_y \) and their derivatives \( \frac{dA_y}{dt} \) are bounded. Consequently, asymptotic stability is obtained if the following conditions are checked [25].

\[ S = a_i + a_6 < 0 \quad \text{(sum of the eigenvalues)} \quad (23) \]
\[ P = a_i a_6 - a_2 a_5 > 0 \quad \text{(product of the eigenvalues)} \quad (24) \]

By choosing two eigenvalues \( \lambda_1 \) and \( \lambda_2 \) located in the left-half of the complex plane, \( S \) and \( P \) are then fixed and the values of the adaptive gains \( \alpha_1 \) and \( \alpha_2 \) are given by solving the following system equation.

\[
\begin{cases}
  a_i + a_6 = S \\
  a_6 a_i - a_2 a_5 = P
\end{cases}
\]

We obtain:

\[
\alpha_1 = -\left(y_{13}K_1 + y_{23}K_3\right)\hat{X} - \frac{S + \sqrt{S^2 - 4(P + a_2 a_5)}}{2} - D
\]

\[
\alpha_2 = \mu_1 + \mu_2 - \frac{S - \sqrt{S^2 - 4(P + a_2 a_5)}}{2} - D
\]

In the model of the estimator of state (3), the only unknown variables are the estimates of the two specific growth rates \( \mu_1 \) and \( \mu_2 \). For that, and according to the expression (4), an estimator of parameters is then necessary.

### 3.2.2. Specific growth rates estimation

The estimates of the growth specific rates will be deduced from the expression (4) (indirect estimation from the estimates of state variables and kinetic parameters). The estimates of the kinetic parameters are correlated with the error estimation of the nitrite and the nitrate concentrations. We propose the following parametric adjustment law:

\[
\dot{\theta} = -\Gamma B_1^T P \tilde{Y}
\]

with: \( B_1 = \begin{bmatrix} -y_{11} H_1^T \hat{X} & 0 \\ y_{12} H_1^T \hat{X} & -y_{22} H_2^T \hat{X} \end{bmatrix} \) and \( \tilde{\theta} = \begin{bmatrix} \hat{\theta_1} \\ \hat{\theta_2} \end{bmatrix} \)

\( P \in \mathbb{R}^{n \times n} \) and \( \Gamma \in \mathbb{R}^{q \times q} \) are arbitrary symmetric positive definite matrices, \( n \) the dimension of \( Y \) and \( q \) the dimension of \( \theta \).
3.3. Stability of the estimator

The above system error (21) can be written as follow:

\[ \dot{Y} = A\dot{Y} + B_i\hat{\theta} \quad (29) \]

By noting that \( \dot{\theta} = \hat{\theta} \) for unknown constant parameters or slow variables, the total system error associated to (28) and (29) can be written in the following form:

\[ \dot{\hat{Y}} = \begin{bmatrix} \dot{\hat{Y}} \\ \dot{\hat{\theta}} \end{bmatrix} = \begin{bmatrix} A & B_i \\ -\Gamma B_i^T P & 0 \end{bmatrix} \begin{bmatrix} \hat{Y} \\ \hat{\theta} \end{bmatrix} \quad (30) \]

For the stability study, the difficulty lies in the choice of the Lyapunov function. We adopted a quadratic Lyapunov function that allows to obtain a derived function as simple as possible. Thus, we consider the following Lyapunov function:

\[ V = \hat{Y}^T P\hat{Y} + \hat{\theta}^T \Gamma^{-1} \hat{\theta} \quad (31) \]

Differentiating \( V \) with respect to time we obtain:

\[
\dot{V} = \dot{\hat{Y}}^T P\hat{Y} + \hat{Y}^T P\dot{\hat{Y}} + \hat{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} + \hat{\theta}^T \Gamma^{-1} \dot{\hat{\theta}}
\]

\[
= \left( \hat{Y}^T A^T + \hat{\theta}^T B_i^T \right) P\hat{Y} + \hat{Y}^T P \left( A\hat{Y} + B_i\hat{\theta} \right) - \hat{\theta}^T P B_i \Gamma^{-1} \hat{\theta}
\]

\[
- \hat{\theta}^T \Gamma^{-1} \Gamma B_i^T P\hat{Y}
\]

\[
= \hat{Y}^T A^T P\hat{Y} + \hat{\theta}^T B_i^T P\hat{Y} + \hat{Y}^T P A\hat{Y} + \hat{Y}^T P B_i \hat{\theta} - \hat{\theta}^T P B_i \hat{\theta} - \hat{\theta}^T B_i^T P\hat{Y}
\]

\[
\dot{V} = \hat{Y}^T \left( A^T P + PA \right) \hat{Y}
\]

(32)

The matrix \( A \) is a Hurwitz matrix, the solution of the matrix equation:

\[ A^T P + PA = -Q < 0 \quad (33) \]

where \( Q \) is an arbitrary positive definite matrix [14, 21, 25]. This implies that:

\[ \dot{V} = -\hat{Y}^T Q\hat{Y} \leq 0 \quad (34) \]

Thus, the total system error (30) is asymptotically stable.

4. Simulation results

The performance of the proposed estimator was tested through simulation studies on a denitrification process in order to estimate the state variables and the kinetic parameters of the process.

The simulation of the model and the estimator is carried out with the initial conditions and the parameters presented respectively in tables 1 and 2. To test the robustness of the estimator with respect to parameters variations, we introduced disturbances on the kinetic parameters of the system and on the dilution rate. These disturbances are presented in table 3. The synthesis parameters of the estimator are shown in table 4.

The results of the application of the considered adaptive estimator on the denitrification process are illustrated in Fig.1 to Fig.13. The evolutions of the nitrate and nitrite concentrations are shown in Fig.1 and Fig.2 respectively. The evolutions of the acetic acid and biomass concentrations and their estimates are given in Fig.3 and Fig.4.
respectively.

Table 1 Initial conditions

<table>
<thead>
<tr>
<th>Variables</th>
<th>Values</th>
<th>Variables</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_1(0) )</td>
<td>0.6 g/l</td>
<td>( \dot{X}(0) )</td>
<td>0.2 g/l</td>
</tr>
<tr>
<td>( S_2(0) )</td>
<td>0 g/l</td>
<td>( \mu_{1\text{max}}(0) )</td>
<td>0.17 h(^{-1})</td>
</tr>
<tr>
<td>( S_3(0) )</td>
<td>2.77 g/l</td>
<td>( \mu_{2\text{max}}(0) )</td>
<td>0.085 h(^{-1})</td>
</tr>
<tr>
<td>( X(0) )</td>
<td>0.15 g/l</td>
<td>( k_{S_i}(0) )</td>
<td>0.05 g/l</td>
</tr>
<tr>
<td>( \dot{S}_1(0) )</td>
<td>0.6 g/l</td>
<td>( k_{S_2}(0) )</td>
<td>0.07 g/l</td>
</tr>
<tr>
<td>( \dot{S}_2(0) )</td>
<td>0 g/l</td>
<td>( k_{S_3}(0) )</td>
<td>0.1 g/l</td>
</tr>
<tr>
<td>( \dot{S}_3(0) )</td>
<td>3 g/l</td>
<td>( D(0) )</td>
<td>0.09 h(^{-1})</td>
</tr>
</tbody>
</table>

Table 2 Parameter values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{1\text{in}} )</td>
<td>1 g/l</td>
<td>( y_{23} )</td>
<td>1.6</td>
</tr>
<tr>
<td>( S_{2\text{in}} )</td>
<td>0 g/l</td>
<td>( \mu_{1\text{max}} )</td>
<td>0.17 h(^{-1})</td>
</tr>
<tr>
<td>( S_{3\text{in}} )</td>
<td>3 g/l</td>
<td>( \mu_{2\text{max}} )</td>
<td>0.085 h(^{-1})</td>
</tr>
<tr>
<td>( y_{11} )</td>
<td>6.2</td>
<td>( k_{S_i} )</td>
<td>0.05 g/l</td>
</tr>
<tr>
<td>( y_{12} )</td>
<td>3.3</td>
<td>( k_{S_2} )</td>
<td>0.07 g/l</td>
</tr>
<tr>
<td>( y_{22} )</td>
<td>1.2</td>
<td>( k_{S_3} )</td>
<td>0.1 g/l</td>
</tr>
<tr>
<td>( y_{13} )</td>
<td>1.1</td>
<td>( k_d )</td>
<td>0.025 h(^{-1})</td>
</tr>
</tbody>
</table>

Table 3 List of parameter changes

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Parameter changes</th>
<th>Time (h)</th>
<th>Parameter Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>( \mu_{1\text{max}} = 0.2 h^{-1} )</td>
<td>350</td>
<td>( D = 0.13 h^{-1} )</td>
</tr>
<tr>
<td>200</td>
<td>( \mu_{2\text{max}} = 0.1 h^{-1} )</td>
<td>400</td>
<td>( k_{S_i} = 0.1 g/l )</td>
</tr>
<tr>
<td>300</td>
<td>( k_{S_i} = 0.2 g/l )</td>
<td>500</td>
<td>( k_{S_i} = 0.17 g/l )</td>
</tr>
</tbody>
</table>
We remark a relatively short time of convergence (50 h). The evolutions of the auxiliary variables $Z_1$ and $Z_2$ are represented in Fig.5 and Fig.6 respectively. We can notice that the time of convergence of these variables was reflected on the estimate of $S_3$ and $X$. Fig.7 to Fig.11 illustrate the evolution of the kinetic parameters of the system.

We note that the biased convergence of these parameters did not deteriorate the estimates of the specific growth rates $\mu_1$ and $\mu_2$ (Fig.12 and Fig.13). This is due to the nonuniqueness of the identification solutions of the kinetic parameters of the growth models (eq.2). Indeed, in accordance with the structure of the models (eq.2), there are several sets of values ($\mu_{1\text{max}}, k_1, k_1$) and ($\mu_{2\text{max}}, k_2, k_1$) leading to the same values of the specific growth rates $\mu_1$ and $\mu_2$.

Apart from the variation appearing on the evolution of the specific growth rate $\mu_2$ at $t=300\, h$ and $t=500\, h$, the variations introduced on the kinetic parameters of the system and the dilution rate did not influence the convergence of the estimator. Besides, the noise of null mean and variance 0.002 added to the measured variables did not influence the behavior of the considered estimator.

In conclusion, according to the various simulation results, we note that the developed nonlinear adaptive estimator presents robustness with respect to the noisy measurement and disturbances introduced.

### 5. Conclusion

In this work, the problem of joint estimation was tackled. In particular, the analysis was focused on the estimation of state variables and kinetic parameters of a denitrification process. In order to estimate the unmeasured states and the unknown kinetic parameters, we developed a joint adaptive estimation algorithm based on the dynamic model of the considered process. This estimator is a combination between an adaptive estimator and an asymptotic observer.

Computer simulations were developed to test the performances of the proposed estimator in the presence of internal disturbances and a noisy measurement. According to the presented results, the effectiveness of this estimator was observed for the reconstitution of the state variables and the specific growth rates in spite of the kinetic parameters convergence to biased values.

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REFERENCES


**Karim Dahech**, Unité de Commande Automatique (UCA) Ecole Nationale d’Ingénieurs de Sfax. E-mail : dahechkarim@yahoo.fr (born on 1977) received the Electrical Engineering Diploma from the Gabes Engineering School, the Master in Automatic and Industrial Data Processing from the Sfax Engineering School in 2004. He joined the Sfax Biotechnology High Institute, as an Associate Assistant of Electric Engineering, since 2004. His research interests are in the areas of modeling, estimation and control of nonlinear system.

**Tarak Damak**, Unité de Commande Automatique (UCA) Ecole Nationale d’Ingénieurs de Sfax (ENIS). E-mail : tarak.damak@enis.rnu.tn received the Electrical Engineering Diploma from the ENIS-Tunisia in 1989, the ‘Diplôme d’Etudes Approfondies’ in Automatic control from the Institut National des Sciences Appliquées de Toulouse-France in 1990, the ‘Doctorat d’Université ’ degree from the Université Paul Sabatier de Toulouse-France in 1994. He is currently a professor in the Department of Mechanical Engineering of the Ecole Nationale d’Ingénieurs de Sfax-Tunisia. His current research interests are in the fields of distributed parameter systems, sliding mode control and observers, adaptive nonlinear control..

**Ahmed Toumi**, Unité de Commande Automatique (UCA), Ecole Nationale d’Ingénieurs de Sfax (ENIS). E-mail: atoumi_06@yahoo.fr (born on 1952) received the Electrical Engineering Diploma from the ENIS, the DEA in Instrumentation and Measurement from University of Bordeaux/France in 1981 and the Doctoral Thesis from the University of Tunis in 1985. He joined the ENIS, as an Associate Professor of Electric Engineering, since 1981. In 2000, he obtained the University Habilitation from the ENIS. The main research area concerns the modeling, the stability of the electric machines, and the electrical networks. He is the President of the international conference on Sciences and Techniques of Automatic control (STA).