A FUZZY CONTROL OF THE INSTABILITIES OF AN AXIAL FLOW COMPRESSOR

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Abstract This paper describes the active control of the instabilities of an axial flow compressor using a fuzzy logic controller. The controller is designed using a genetic algorithm. The control variable is the throttle of the outlet valve, situated after the compressor. Simulation results show that the controller is able to control both the surge and the rotating stall in the critical operating point, thus increasing the region of stability.

Key Words, Compressor; rotating stall; surge; fuzzy controller; genetic algorithms

1. Introduction
Several theoretical and experimental studies have shown that the active control can suppress the aerodynamic instabilities of the axial flow compressor. The main incentive of these studies is that the active control permits to increase the region of stability and therefore to improve the performances of the compressor. The characteristic of compression shows that the maximum of pressure delivered by the compressor is located to the limit of the stability ([1],[2]). In this critical operating point, disturbances such as the acceleration of the motor and variations in the inlet are capable of destabilizing the compressor giving rise to surge and rotating stall. In this low-pressure air pockets create at the circumference of the compressor and turn around its axis of rotation [2].

The objective of the active control is to allow for maximal compression while consolidating in a robust manner the operation around the critical point by modifying the dynamics of the compressor through closed loop control. This is achieved by increasing the region of stability beyond the so-called line of surge. Several approaches have been proposed for active control. These vary according to the sensors, the actuators and the algorithms of control. Most control algorithms control are based on the non linear control to see for example ([1],[9]). From a technological point view, the actuators can be the bleed valves [9], or gas injectors, placed at the circumference of the compressor [8] or the throttle of the outlet valve ([2], [4], [5]).

In this paper, we propose a method of control of the instabilities of the compressor using fuzzy logic controller. The objective is to eliminate both surge and rotating by manipulating the throttle of the outlet valve. The controller is designed using a genetic algorithm. The design strategy is based on a closed loop simulation for a nominal model with the controller in the loop. The simulation is based on the model of Moore and Greitzer [10] which is used in most studies published during the last decade use ([1], [9]), because of its remarkable capacity to describe qualitatively and to predict the behaviour of the compressor. After the controller is designed, it is tested for robustness operating conditions other than the nominal.

This paper is organised as follows: section 2 introduces briefly the model of Moore and
Greitzer, section 3 present the design of the fuzzy controller by the genetic algorithms while section 4 the results of simulation.

2. The model of the compressor
The three-dimensional model of Moore and Greitzer is given by the following ordinary differential equations [1]:

\begin{align}
\dot{\Psi} &= \frac{1}{\beta^2} (\Phi - \Phi_T(\Psi)) \\
\dot{\Phi} &= -\Psi + \Psi_c(\Phi) - 3\Phi R \\
\dot{R} &= \sigma R (1 - \Phi^2 - R) \quad \sigma > 0 \\
\Psi_c(\Phi) &= \Psi_{c0} + 1 + 1.5\Phi - 0.5\Phi^3 \\
\Phi_T &= \sqrt{\gamma} \Psi - 1, \gamma > 0
\end{align}

\(\Phi\) is the annulus averaged mass flow rate, \(\Psi\) the pressure rise and \(R\) the square amplitude of the rotating stall cell. \(\Psi_c(\Phi)\) and \(\Phi_T\) represent the characteristics of the compressor and that of the outlet valve with \(\gamma\) a parameter of the throttle. The parameter \(\Psi_{c0}\) is related to the nature of the compressor and \(\beta\) a parameter function of the plenum volume and the wheel speed. It is admitted that the parameter \(\gamma\) can be decomposed as \(\sqrt{\gamma} = (u + \mu)\), where \(u\) is proportional to the section of the bleed valve and used as actuator, and \(\mu\) a parameter representing the effect of disturbances from inlet and combustion chamber [1].

The model described by the equations (1)-(5) can be rewritten in the following form:

\[
\begin{bmatrix}
\dot{R} \\
\dot{\Phi} \\
\dot{\Psi}
\end{bmatrix} = f(R, \Phi, \Psi) + g(R, \Phi, \Psi)(u + \mu),
\]

with

\[
f(R, \Phi, \Psi) = \begin{bmatrix}
\sigma R (1 - \Phi^2 - R) \\
-\Psi + \Psi_c(\Phi) - 3\Phi R \\
(\Phi + 1)/\beta^2
\end{bmatrix}
\quad \text{and} \quad
g(R, \Phi, \Psi) = \begin{bmatrix}
0 \\
0 \\
-\sqrt{\Psi}/\beta^2
\end{bmatrix}
\]

The equilibrium set \(E\) of the control system is given by

\[
f(R, \Phi, \Psi) + g(R, \Phi, \Psi)(u_e + \mu) = 0
\]
An axisymmetric equilibrium point is a point in $E$ with $R = 0$ and a nonaxisymmetric equilibrium point describing the rotating stall is a point where $R > 0$.

Considering as usual all variable dimensionless ([1], [2], [3]), we have:

if $R_e = 0$

$$\Psi_e = \Psi_e(\Phi_e)$$ \hspace{1cm} (9)

$$(u_e + \mu) = (\Phi_e + 1) / \sqrt{\Psi_e}, \quad \Phi_e \in R$$ \hspace{1cm} (10)

and if $R_e = 1 - \Phi_e^2$

$$\Psi_e(\Phi_e) = \Psi_{e0} + 1 + 1.5\Phi_e + 2.5\Phi_e^3$$ \hspace{1cm} (11)

$$(u_e + \mu) = (\Phi_e + 1) / \sqrt{\Psi_e}, \quad -1 < \Phi_e < 1$$ \hspace{1cm} (12)

The maximum of pressure rise is obtained at the critical point ($\Phi_0 = 1$, $\Psi_0 = \Psi_{e0} + 1$ and $R_0 = 0$)

Introducing the new variables:

$$e_\Phi = \Phi - \Phi_e, \quad e_{\Psi} = \Psi - \Psi_e, \quad e_R = R - R_e$$ \hspace{1cm} (13)

$$e_u = u - u_e$$ \hspace{1cm} (14)

Around an equilibrium point $O(R_e, \Phi_e, \Psi_e, u_e)$, the model can be written in the following form [1]:

$$\begin{bmatrix}
\frac{d(e_R)}{dt} \\
\frac{d(e_\Phi)}{dt} \\
\frac{d(e_{\Psi})}{dt}
\end{bmatrix} = f(R, \Phi, \Psi) + g(R, \Phi, \Psi)(e_u + u_e + \mu)$$ \hspace{1cm} (15)

This model is used in the following section to design the fuzzy logic controller.

3. **The fuzzy logic controller**

Fuzzy logic controllers have become a classical tool for the control engineer. A Fuzzy Logic Controller (FLC) based on the Mamdani inference system is described by a set of fuzzy rules of the type [11]:

$R_i : \text{if } X_1 \text{ is } A_i \text{ and } X_2 \text{ is } B_i \text{ then } U \text{ is } C_i$

Where $R_i$ is the $i^{th}$ rule and $A_i, B_i, C_i$ are respectively the fuzzy set associated with the variables linguistic variables $X_1, X_2$ and $U$. The fuzzy sets are characterized by membership functions. A fuzzy controller is defined completely by its structure and its parameters. The structure is determined by the input and output variables of the controller, the number of fuzzy set associated with each variable and the shape of the membership functions. The parameters define the distribution of the membership functions on the universe of discourse and the rule base. An FLC can be obtained from the expertise of an operator or designed using a model. Several methods have been
developed for the design of an FLC, the most popular are those based on the genetic algorithms [12].

Genetic algorithms, GA, developed by John Holland are heuristics based on the process of natural evolution. They simulate this process through coding and special operators. They work with a population of chromosomes or individuals. A chromosome is composed of subchromosomes each representing a coding of one variable of the problem and each chromosome represents a feasible solution to the problem with an associated value of the cost function or fitness to be optimised. Binary coding of the variables has been widely used. A population is a set of feasible solutions. By applying genetic operators to the current population a new population is created with the goal of improving the fitness. Simple GA use three operators: reproduction, selection and mutation.

GA has been used diversely for FLC design. This is achieved through closed loop simulation. The search is carried out either in the set of the parameters of the membership functions and/or in the set of rules [13].

In this work, we use a simple GA [12] to design the fuzzy logic controller. The structure of the FLC and the rule base are chosen by the designer, the genetic algorithm is used for finding the distribution of the membership functions on the universe of discourse that gives a stable closed loop system. The solution methodology is based on a closed loop simulation with the controller in the loop. For every generated chromosome, the controller is constructed and placed in the loop, a simulation for a sufficiently long time so as to identify unstable solutions is carried out. The cost function is determined then by the cumulated error. A very high cost will be affected to the unstable solutions.

4. Simulation and discussion

4.1. Design of the FLC

The FLC has two input variables $e_\Phi$, $e_\Psi$ and one output variable $e_u$ as defined in (13) and (14). The membership functions of these variables noted $\mu(e_\Phi)$, $\mu(e_\Psi)$ $\mu(e_u)$ are represented in Fig 1.

![Fig1](image-url)  
Fig1. Membership functions $\mu(e_\Phi)$, $\mu(e_\Psi)$ $\mu(e_u)$

The rule base of the FLC is symmetric and is given in table 1.
The distribution of the membership functions on the universes of discourse of the different variables $e_\Phi$, $e_\Psi$, and $e_u$ is determined by the genetic algorithms. The objective function is given by the cumulated error on the rise of pressure and that is

$$J = \sum T (e_\Phi^2 + e_\Psi^2),$$  \hspace{1cm} (16)$$

with $T$ the simulation time.

The genetic algorithm was run for 100 generations, the number of individuals per population was set to 50. The initial population was generated randomly. The parameters obtained by the genetic algorithm correspond to the minimal value of the criteria in the population of the last generation. These are $a_1 = -0.6$, $b_1 = 0$, $c_1 = +0.6$, $a_2 = -3$, $b_2 = 0$, $c_2 = 3$, $a_3 = -0.6$, $b_3 = 0$, $c_3 = +0.6$. The parameters $a_i$, $b_i$, and $c_i$ are as defined in Fig 1.

The parameters used in simulation, are dimensionless as in [1]:

$$\lambda = 1.256, l_c = 21.67, B = 0.2, a = 0.1, H = 0.0616, W = 0.1341, \Psi_{c0} = 0.1469$$

$$\sigma = \frac{3\lambda l_c}{1 + \alpha \lambda} = 5.7756, \beta = \frac{3B_\Phi}{W} = 0.1837$$

The controller is designed for the nominal case in a nonaxisymmetric equilibrium point $\Phi_e = 0.5$ and the initial conditions, $\Phi_0 = 0.7$, $R_0 = 0.45$, $\Psi_0 = 0.4$. Fig. 2 shows that the system is brought to the desired equilibrium point and the three variables $e_\Phi$, $e_\Psi$, and $e_R$ go to zero and Fig. 3 shows that the control signal goes to equilibrium value.

### 4.2 Testing of the FLC

In the following we test the behaviour of the FLC for situations other than the nominal case.

**Test 1:**

The designed FLC is tested for the non axisymmetric equilibrium point $\Phi_e = -0.5$ and the initial conditions, $\Phi_0 = 0.7$, $R_0 = 0.45$, $\Psi_0 = 0.4$. Fig. 4 shows that the controller succeeds in controlling the system and that the control signal $u$, represented in Fig. 5, converges toward the equilibrium value $u_e$.

**Test 2:**

In this second example the controller is tested for the apparition of both surge and rotating stall. In this simulation, the parameters are chosen equal to $\Psi_{c0} = 2.5$, $\beta = 0.6698$ and $\mu = 0.8525$, with the initial conditions $\Phi_0 = 0.8$, $R_0 = 0.75$, $\Psi_0 = 2.9$. From $t=0$ to $t=50$ the fuzzy controller is switched off and both surge and rotating stall appear. At time $t = 50$, the fuzzy controller is applied to the system and the variables $e_\Phi$, $e_\Psi$, and $e_R$ are taken to their equilibrium values with elimination of the surge and rotating stall. Fig. 6 represents the result of a simulation that shows the controller’s faculty to control simultaneously the

### Table 1

The rule base of the FLC

<table>
<thead>
<tr>
<th>$e_\Phi$</th>
<th>$e_\Psi$</th>
<th>Small(S)</th>
<th>Medium(M)</th>
<th>Big(B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small(S)</td>
<td>B</td>
<td>B</td>
<td>M</td>
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</tr>
<tr>
<td>Medium(M)</td>
<td>B</td>
<td>M</td>
<td>S</td>
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<td>Big(B)</td>
<td>M</td>
<td>S</td>
<td>S</td>
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surge and the rotating stall to the critical operating point defined. Thus, under the action of controller FLC, the system stabilizes to the axisymmetric equilibrium point $\Phi_e = 1, \text{Re} = 0, \Psi e = \Psi_{e0} + 2$ and the instabilities are eliminated. Fig. 7 represents the control signal variations.

**Fig. 2.** Nominal case: output variables errors, $e_\Phi : \text{---}$, $e_\Psi : \text{----}$, $e_R : \text{--}$. 

**Fig. 3.** Nominal case, the control signal
Fig 4. Test 1 Output variables errors \( e_\Phi , e_\Psi , e_R \).

Fig 5. Test 1 the control signal.

Fig 6. Test 2, the output variables errors: \( e_\Phi , e_\Psi , e_R \).
5. Conclusion
In this article, we proposed an approach for controlling the instabilities of the compressor using fuzzy logic controller. The controller uses two easily measurable variables and the throttle of the outlet valve as the control signal. The rule base of the controller is symmetric and the number of fuzzy sets is defined by user, the distribution of the membership functions is determined using a genetic algorithm. Simulation results show that the controller is robust and manages to eliminate both the surge and rotating stall. In comparison with the existing results, the fuzzy logic controller behaves as well as other control laws presented in the literature.

REFERENCES

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