EVALUATION OF THE RADIATIVE WAVE PROPAGATION EFFECT IN EMT

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Abstract Currently, modelling of EMT systems does not take into account all the electrodynamic effects. While only considering eddy current effects is adequate for particular industrial EMT systems where high conductivity materials are normally encountered, it is not certain whether the radiative wave propagation effect can be ignored for biomedical or low conductivity EMT systems which use high excitation frequencies and in particular, for imaging materials with high permittivity (such as water), where signal from the radiative field can be significant compared to that from the diffusive field. This paper uses an exact analytical solution for a coil(s) near a stratified media derived from a full wave theory to evaluate the effect of eddy currents and wave propagation at a range of frequencies. This allows a better physical insight from which the highest frequency can be determined at which EMT can make reliable eddy current measurements without considering the wave propagation effect.

Key Words, Electromagnetic Tomography, eddy currents, wave effect, analytical solution

1. Introduction

EMT (Electromagnetic Tomography or Electromagnetic Induction Tomography, also termed as Mutual Inductance Tomography) has developed during the last decade for industrial process applications (e.g. visualising industrial processes such as those in metal production [1]) and for biomedical applications (e.g. determining body composition [2], imaging human thorax and head [3] and imaging brain oedema [4] etc). A set of coils are employed and distributed around the object being imaged and measurements of mutual inductive coupling between the coils are taken and used for image reconstruction. The inductive coupling between the coils changes as the objects are subject to eddy current effects and generate secondary fields. Generally, the magnetic fields can be described in phasor notation and can be explained as follows. An excitation coil generates a sinusoidal magnetic field, which in turn induces eddy currents in the object. In the low conductivity limit, these induced currents are proportional to the object conductivity and are 90° out of phase with respect to the excitation current. Hence, a secondary magnetic field that is 90° out of phase with respect to the excitation current is produced by the eddy currents. This secondary magnetic field generates a voltage at the receiving coils that is 180° out of phase with the excitation current. Therefore, in low conductivity EMT, the resistive components instead of inductive components are measured and used for image reconstruction. Currently, modelling of EMT only considers eddy current effects, i.e. solving diffusion equations, instead of full wave equations, possibly due to the significant complexity and the increased computation requirements of the latter [5]. Although considering only eddy currents is adequate for industrial applications targeting high

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conductivity materials, the lack of understanding of the wave propagation effects in relation to the eddy current effects poses a difficulty for applying EMT to low conductivity processes (such as multi-phase flow) and increasing the excitation frequency and ultimately sensitivity.

This paper aims to evaluate the influence of wave effects in EMT using an analytical solution.

2. Model and Theory

A complete analytical solution for a full three dimensional model of EMT without any simplification is extremely complex, difficult to obtain and computationally intensive. The aim of this paper is not to find such a model, but to evaluate the radiative wave propagation delay effect in low conductivity EMT in order to understand the influence of the radiative wave propagation delay effect and determine the upper frequency limit for making reliable eddy current measurements. The following assumptions are made so that a relatively simple model can be obtained.

1. The coils are circular and filamentary.
2. Homogenous media are considered.
3. The size of the coils is much smaller than the imaging space, and therefore the imaging space can be assumed as being flat and parallel to the coil surface.
4. Only the self-impedance of the excitation coil and mutual-impedance of the opposite coils are considered because these two case represent the two extremes of the radiative wave delay effects.

Based on above assumptions, the model in Figure 1 was used to represent the simplified EMT.

![Figure 1 Simplified EMT model for evaluating the wave propagation delay effects](image)

To evaluate the wave propagation delay in relation to the eddy current effects, we
consider two solutions of the model. The first solution calculates the voltage sensed on the receiving coil when only eddy current effects are considered. The second solution takes into account both eddy current effects and wave propagation delay effects. Then the results from these two solutions can be compared and the error due to neglecting the wave propagation delay effects can be quantified.

We start from the magnetic vector potentials in regions I, II and III shown in Figure 1. Considering only the eddy current effects, the magnetic vector potentials in each region are as follows [6].

\[
A^{(1)}(r, z) = \frac{\mu_0 I_r}{2} \int_0^\infty J_1(\alpha r_0) J_1(\alpha r) e^{-\alpha_0 |z|} \frac{\alpha d\alpha}{\alpha_0}
\]

\[
+ \frac{\mu_0 I_r}{2} \int_0^\infty J_1(\alpha r_0) J_1(\alpha r) e^{-(\alpha_1+\alpha_0)z} \frac{(\alpha_0 + \alpha_1)(\alpha_1 - \alpha_2) + (\alpha_0 - \alpha_1)(\alpha_1 + \alpha_2) e^{2\alpha c}}{(\alpha_0 - \alpha_1)(\alpha_1 - \alpha_2) + (\alpha_0 + \alpha_1)(\alpha_1 + \alpha_2) e^{2\alpha c}} \frac{\alpha d\alpha}{\alpha_0}
\]

\[
A^{(2)}(r, z) = \mu_0 I_r \int_0^\infty J_1(\alpha r_0) J_1(\alpha r) e^{-\alpha_0 r} \frac{(\alpha_0 + \alpha_2) e^{2\alpha c} e^{\alpha_2 z} + (\alpha_0 - \alpha_2) e^{-\alpha_2 z}}{(\alpha_0 - \alpha_1)(\alpha_1 - \alpha_2) + (\alpha_0 + \alpha_1)(\alpha_1 + \alpha_2) e^{2\alpha c}} \frac{\alpha d\alpha}{\alpha_0}
\]

\[
A^{(3)}(r, z) = \mu_0 I_r \int_0^\infty J_1(\alpha r_0) J_1(\alpha r) e^{-\alpha_0 r} \frac{2\alpha_0 e^{(\alpha_0 + \alpha_2)z} e^{\alpha_2 z}}{(\alpha_0 - \alpha_1)(\alpha_1 - \alpha_2) + (\alpha_0 + \alpha_1)(\alpha_1 + \alpha_2) e^{2\alpha c}} \frac{\alpha d\alpha}{\alpha_0}
\]

Where \( \mu_0 \) denotes the permeability of free space, while \( l \) denotes the height of the bottom of the excitation coil; and \( c \) denotes the thickness of the media.

\( \alpha_0 = \alpha ; \alpha_1 = \sqrt{\alpha^2 + j \sigma \mu_1 \omega} ; \alpha_2 = \alpha \cdot J_f(x) \) is a first order Bessel function of the first kind. \( I \) is the current flowing through the excitation coil. \( r_0 \) is the radius of the coils.

Therefore, the voltage induced in the excitation coil can be expressed as:

\[
V^{(1)} = j\omega 2\pi r_0 A^{(1)}(r_0, z) = j\omega 2\pi r_0^2 \mu_0 I \int_0^\infty J_1(\alpha r_0) J_1(\alpha r_0) e^{-\alpha_0 |z|} \frac{\alpha d\alpha}{\alpha_0}
\]

\[
+ j\omega 2\pi r_0^2 \mu_0 I \int_0^\infty J_1(\alpha r_0) J_1(\alpha r_0) e^{-(\alpha_1+\alpha_0)z} \frac{(\alpha_0 + \alpha_1)(\alpha_1 - \alpha_2) + (\alpha_0 - \alpha_1)(\alpha_1 + \alpha_2) e^{2\alpha c}}{(\alpha_0 - \alpha_1)(\alpha_1 - \alpha_2) + (\alpha_0 + \alpha_1)(\alpha_1 + \alpha_2) e^{2\alpha c}} \frac{\alpha d\alpha}{\alpha_0}
\]

Where the first item corresponds to the coil voltage induced by the primary field in the absence of the media in the imaging space (i.e. coil voltage in the air), and the second item is the induced voltage due to the media in the imaging space, which is of interest to us.

\[
\Delta V^{(1)} = j\omega 2\pi r_0^2 \mu_0 I \int_0^\infty J_1(\alpha r_0) J_1(\alpha r_0) e^{-(\alpha_1+\alpha_0)z} \frac{(\alpha_0 + \alpha_1)(\alpha_1 - \alpha_2) + (\alpha_0 - \alpha_1)(\alpha_1 + \alpha_2) e^{2\alpha c}}{(\alpha_0 - \alpha_1)(\alpha_1 - \alpha_2) + (\alpha_0 + \alpha_1)(\alpha_1 + \alpha_2) e^{2\alpha c}} \frac{\alpha d\alpha}{\alpha_0}
\]

Similarly, the voltage induced in the receiving coil

\[
V^{(3)} = j\omega 2\pi r_0^2 \mu_0 I \int_0^\infty J_1(\alpha r_0) J_1(\alpha r_0) e^{-\alpha_0 r} \frac{2\alpha_0 e^{(\alpha_0 + \alpha_2)z} e^{\alpha_2 z}}{(\alpha_0 - \alpha_1)(\alpha_1 - \alpha_2) + (\alpha_0 + \alpha_1)(\alpha_1 + \alpha_2) e^{2\alpha c}} \frac{\alpha d\alpha}{\alpha_0}
\]
The voltage in the receiving coil due to the primary field can be obtained by assigning \( \alpha_2 = \alpha_1 = \alpha_0 \)

\[
V_{air}(3) = j\omega 2\pi \varepsilon_0^2 \mu_0 I \int_0^\infty J_1(\alpha r_0) J_1(\alpha r_0) e^{-\alpha z} \frac{\omega \varepsilon_0}{2\alpha_0} \frac{\alpha d\alpha}{\alpha_0} \]

And therefore the induced voltage due to the media in the imaging space for the opposite receiving coil is

\[
\Delta V^{(3)} = j\omega 2\pi \varepsilon_0^2 \mu_0 I \int_0^\infty J_1(\alpha r_0) J_1(\alpha r_0) e^{-\alpha z} \frac{2\alpha_0 e^{(\alpha \varepsilon_0 + \omega \mu_2^2) e^{\alpha z}}}{(\alpha_0 - \alpha_1)(\alpha_1 - \alpha_2) + (\alpha_0 + \alpha_1)(\alpha_1 + \alpha_2) e^{2\alpha_0}} \frac{\omega \varepsilon_0}{2\alpha_0} \frac{\alpha d\alpha}{\alpha_0} \]

Considering the wave propagation delay effects in addition to the eddy current effects, equations (1-8) take the same formats except [7]

\[
\alpha_0 = \sqrt{\alpha_1^2 - k_0^2} \\
\alpha_1 = \sqrt{\alpha_2^2 - k_1^2} \\
\alpha_2 = \sqrt{\alpha_2^2 - k_2^2}
\]

Where \( k_0, k_1, \) and \( k_2 \) are respectively the propagation constants in region I, II and III.

\[
k_0^2 = \varepsilon_0 \mu_0 \omega^2 \\
k_1^2 = \varepsilon_1 \mu_1 \omega^2 - j \sigma \mu_1 \omega \\
k_2^2 = \varepsilon_2 \mu_2 \omega^2
\]

The wave effects enter the equations through the items \( \varepsilon_0 \mu_0 \omega^2 \), \( \varepsilon_1 \mu_1 \omega^2 \) and \( \varepsilon_2 \mu_2 \omega^2 \). Note we assume regions I and III are not conductive and therefore without eddy current effects.

3. Results and analysis

In the simulation, the coil radius is set to 5cm and the distance between the excitation coil and the opposite coil is set to 1m. Unit current was applied to the excitation coil.

Figure 2 shows the induced voltages on the excitation coil due to the media when taking account of the wave propagation effects and when considering the eddy current effects alone. It can be seen that (1) as the conductivity of the object decreases, the deviation between models 1 (eddy current alone) and model 2 (full wave effects) increases because wave propagation delay effects become more predominant compared with eddy current effects, i.e., \( \omega \varepsilon > \sigma \) (see figure 3); (2) as frequency increases, the deviation becomes larger for a similar reason in addition to the fact that the wavelength becomes smaller (see figure 4), which increases the wave delay effects.
Figure 2 the induced voltages of the excitation coil at different frequencies for saline solutions (a) $\varepsilon=81, \sigma=0.01$; (b) $\varepsilon=81, \sigma=0.1$; (c) $\varepsilon=81, \sigma=1$ using solutions 1 and 2

Figure 3 Ratio between conduction current and displacement current ($\varepsilon=81$)
Figure 4 Wavelength for water with different conductivities ($e=81$)
Figure 5 the induced voltages of the opposite receiving coil at different frequencies for saline solutions (a) $\varepsilon=81$, $\sigma=0.01$; (b) $\varepsilon=81$, $\sigma=0.1$; (c) $\varepsilon=81$, $\sigma=1$ using solutions 1 and 2
In the case of the opposite coil, Figure 5 shows the induced voltages in the receiving coil due to the media when taking account of the wave propagation effects (solution 2) and when considering the eddy current effects alone (solution 1). The same two conclusions drawn from the case where the induced voltage on the excitation coil was considered are true here. In addition, the effects of wave delay are generally more pronounced for the opposite receiving coil as the EM wave needs to travel further to reach the opposite coil and the phase change resulting from this propagation delay has a more pronounced effect.

![Graph](image)

**Figure 6** The highest frequency for the error due to neglecting the radiative wave propagation effect to be within 10%

One of the aims of this paper is to determine the highest frequency at which the EMT can make reliable eddy current measurements without considering the wave propagation effect. Therefore, for each case, we identify the frequency when the signal from the radiative field accounts for 10% percent of the whole signal, i.e. the error due to neglecting the radiative wave propagation effect reaches 10% (Figure 6). It can be seen that for opposite coil, the highest frequency for water with a conductivity of 0.01 is slightly less than 1MHz in order for the error caused by wave delay effect to be within 10%. In general, the highest frequencies are in the range of 1MHz to 40MHz for the simulated cases. In those frequencies, the wavelengths are comparable to the imaging space dimension and the displacement currents are in a similar order with the eddy currents.
4. Conclusions

This paper used an analytical solution derived from a full wave theory to evaluate the effect of eddy currents and wave propagation at a range of frequencies. Better physical insight are gained into the EM effects in EMT and the highest frequency at which the EMT can make reliable eddy current measurements without considering the wave propagation effect are determined for the cases considered.

References


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