MULTI-INCENTIVE STRATEGY IN MULTI-SERVICE BASED NETWORKS

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Abstract. A Lotka-Volterra like equation of the state of a multi-user dynamic network system is established. An analytical model of the traffic pricing of the multi-user two-priority networks is presented and the concept of Stackelberg incentive strategy in the game theory is applied into the network system. Non-linear incentive strategies are proposed. By means of the incentive scheme and the re-assignment of the traffic of the network system, the network users and network managers can get the best surplus, at the same time, the system gets to an ideal and steady position.

Key Words. pricing; incentive strategy; priority; game theory

1. Introduction

With the rapid development of the Internet, awareness of the need for high-speed integrated networks has reached the mainstream. Network traffic has grown exponentially in recent years, and there seems to be no dispute that this trend will continue in the future. Spurred by not meeting the demand of users, engineers roll out applications with ever increasing resource requirements. At the same time, the user base will increase. These forecasts imply a scarcity of network resources (e.g., buffer, bandwidth, and processing). Pricing can thus be used as means to resolve the problem of allocating these scarce resources to users. Multiple-service networks require incentives for users to choose the service that is most appropriate to their needs, thereby discouraging excess allocation of resources and maximizing statistical multiplexing capabilities. This is most commonly achieved through pricing. Networking pricing mechanisms have recently received much attention as a technique for traffic management and congestion control within multi-service networks. The current debates on how to charge for Internet services [1] and ATM services [2–4] have sparked renewed interest in the former network pricing. Pricing network resources as a method to maximize the usage of network links and as a way of supporting priority based on traffic have been a topic of research for many years [5–7]. Cocchi [8] proposed a priority pricing scheme for multiple service disciplines in computer networks. In [9], the author provided an overview of pricing concepts for broadband multi-service networks. [10] presented a fully distributed simple yet efficient pricing mechanism that combines controlling QoS traffic with pricing policies for both best effort and QoS traffic. Using readily available network measurements, it was shown that how near-optimal pricing policies could be obtained for resource allocation between QoS and best-effort traffic. Further, the pricing and billing portions of the scheme can be implemented with minimal overhead. In
[11 ~ 15], authors discussed static pricing policies for multi-service networks. It demonstrated that how, by adopting an appropriate policy, the service provider is able to offer the needed incentives for each user to choose the service that best matched his (her) needs, thereby discouraging over allocation of resources and improving social welfare. Furthermore, by indirectly revealing their QoS or resource requirements, users provide information of great value to the traffic management task.

In this paper, we investigate the dynamic equilibrium problem of the network system by means of the differential equations theory. Using the concepts of Nash equilibrium and Stackelberg incentive strategy in the game theory, the incentive pricing strategy is devised for the management problem. In the ideal state, the nonlinear and crossing influence incentive pricing strategy is presented to encourage and guide the noncooperative users to select the serving request, which is helpful for the whole network, intensifying the cooperation of the users and the network, also increasing the usage of network resource rate by using the instantaneous change of rate on network and deviating from the state equilibrium point.

2. Model Description

2.1. A multi-user dynamic network system model

Consider a multi-user two-priority networks system. Let $S_i$ be the set of choices available to user $i$ when it requests service from the network. The joint strategy space, denoted by $E$, is the Cartesian product of the individual strategy sets; for $N$ users, we have:

$$E = S_1 \times S_2 \times \cdots \times S_N = \{S = (s_1, s_2, \cdots, s_N)^T : s_i \in S_i\}.$$  

Let $s_h$ be the set of choices available to the traffic as high priority for all customers. We get:

$$s_h = s_1 + s_2 + \cdots + s_N$$

Because the multi-user two-priority networks system differs from the dynamics of the quality function$^{[12]}$, we establish a Lotka-Volterra like equation of the state of a multi-users dynamic network system. More specifically,

$$\frac{\partial s_h}{\partial t} = s_h f(s_h) = s_h(a_0 - a_1 s_h),$$

where $a_0 > 0$, $a_1 > 0$.

System (3) is stable at the equilibrium point $M^*(s^*_h) = M^*(\frac{a_0}{a_1})$.

2.2. Utility functions of the multi-users network system

Utility functions serve to quantify the tradeoffs that customers are willing to make between the quality of a service received and its price. We consider that each individual user associates a value to each service level; this value, referred to as the user’s utility function denoted $U_i(q_i)$, can be interpreted as the amount the user is willing to pay for a given QoS.

Surplus function of user $i$, $c_i(s_i, s_{-i})$, is defined as the difference between the utility obtained with a given service choice and the price paid for the service $V(P)$. This relationship can be expressed as$^{[13]}$:

$$c_i(s_i, s_{-i}) = U_i(q_i) - V_i(P).$$

Each user can choose to tag a percentage $s_i$ of his (her) traffic as high priority, paying a price $p_h$; the remainder of the traffic is transmitted as low-priority at a price
Due to the scarcity of closed-form results for delay in G/G/1 priority queuing systems, we assume a Poisson arrival process for the queue, with 
\[ t_i = (\lambda_i, \bar{x}_i, x^2_i) \]
where \( \lambda_i \) is the average arrival rate, and \( \bar{x}_i \) and \( x^2_i \) are the first two moments of message length for user \( i \). For the sake of concreteness, quality of service in this system will be measured by average waiting time in the queue, denoted by \( w_i \).

Assume:
\[ \bar{x}_i = \bar{x}, x^2_i = \bar{x}^2. \]

Surplus functions of the user \( i \) will be approximated by\(^{[13]}\):
\[ c_i(s_i, s_{-i}) = \frac{A_i - B_i w_i^{d_i}}{U_i(q_i)} - \frac{\Delta p_i \lambda_i - p_i \lambda_i}{V_i(p_h, p_l)} \quad (i = 1, 2, \cdots, N), \]
where
\[ w_i = k \frac{\bar{x}^2 - \bar{x}^2_i}{1 - \bar{\lambda}_T x}, \]
\[ \lambda_T = \lambda_1 + \lambda_2 + \cdots + \lambda_N. \]

Parameters \( A_i, B_i \) and \( d_i \) are not arbitrary. Assume:
\[ S = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_N \end{pmatrix}, \quad C(S) = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{pmatrix}, \quad A = \begin{pmatrix} A_1 \\ A_2 \\ \vdots \\ A_N \end{pmatrix}, \]
\[ B = \begin{pmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_N \end{pmatrix}, \quad H = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \end{pmatrix}, \quad W = \begin{pmatrix} w_1^{d_1} \\ w_2^{d_2} \\ \vdots \\ w_N^{d_N} \end{pmatrix}, \]
\[ T = \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_N \end{pmatrix}, \quad \Delta p = p_h - p_l. \]

System (5) can thus be expressed as:
\[ C(S) = A - BW - \Delta pTS - p_l H. \]

2.3. Nash equilibrium of the system at an ideal state

A Nash equilibrium is a strategy combination where no user can unilaterally increase his (her) utility by changing his (her) strategy. More precisely, we introduce the following definition\(^{[13]}\):

**Definition 1 (Nash Equilibrium)** Strategy combination \( S^{*} \) is a Nash equilibrium if:
\[ c_i(S^{*}) \geq c_i(s_i, s^*_{-i}), \forall s_i \in S_i, i \in (1, 2, \cdots, N). \]

If an equilibrium exists, we would like to determine whether it is efficient; for this purpose, we use the concept of Pareto optimality. A strategy combination is Pareto optimal if there is no other strategy combination which at least one user would prefer and from which all others would be different; more formally\(^{[14]}\):
Definition 2 (Pareto Optimality) A strategy combination $S^*$ is Pareto optimal if there does not exist $S \in S$ such that:
1. $c_i(S') \geq (c_i(S^*), \forall i$;
2. $c_i(S') > (c_i(S^*)$, for at least one $i$.

If the network system achieves a Nash equilibrium in the state equilibrium, the network system is in the ideal state expected by the network providers and users. We propose the following definition:

**Definition 3 (Ideal State)** The network system is in the ideal state if the network system arrives at a Nash equilibrium in the state equilibrium.

2.4. The Nash equilibrium condition

In the simple case of two-user with identically independently distributed arrivals to the queue, and with exponentially distributed message lengths of mean $\mu$ and constant marginal utility $B_i$, we are able to determine necessary and sufficient conditions on $\Delta p$ for the existence of an optimal equilibrium.

Lemma Under the assumptions listed above, a two-user system achieves a unique Nash equilibrium that is Pareto optimal and maximizes revenue if and only if:

$$
\min_i B_i \left( \frac{2\lambda}{\mu - 2\lambda}(\mu - \lambda) \right) < \Delta p < \max_i B_i \left( \frac{2\lambda}{\mu - 2\lambda}(\mu - \lambda) \right).
$$

If $\Delta p$ is not outside the interval described in (8), network system can’t achieve the Nash equilibrium that is Pareto optimal and maximizes revenue. In the following, Stackelberg incentive strategy in the game theory is applied into the network system. The Incentive strategy make the network system achieve Nash equilibrium in the ideal state.

3. Incentive strategy of multi-user tow-priority network system

In order to ensure the network system (5) to have an equilibrium point $S^* = (s_1^*, s_2^*, \ldots, s_N^*)$ in the ideal state, which is Pareto optimal, where

$$
s_1^* + s_2^* + \cdots + s_N^* = s_h^*,
$$

we take two appropriate policies $P_1$ and $P_2$ to system (7). Let:

$$
C(S, P_1, P_2) = A - BW - \Delta p TS - pH - P_1 - P_2,
$$

where

$$
P_1 = (p_{11}, p_{21}, \cdots, p_{N1})^T,
$$

and

$$
P_2 = (p_{12}, p_{22}, \cdots, p_{N2})^T.
$$

$p_{1i}$ is paid additionally by the user $i$ if $s_i$ departs from the Nash equilibrium point of system (5). $p_{2i}$ is paid additionally by the user $i$ if $s_h$ departs from the Nash equilibrium point of system (5).

The surplus function of the network system will be described as:

$$
C_0(S, P_1, P_2) = (\alpha_1, \alpha_2, \cdots, \alpha_N)C(S, P_1, P_2),
$$

where

$$
\alpha_1 + \alpha_2 + \cdots + \alpha_N = G(\alpha_i > 0),
$$
with \( G \) being constant.

Therefore, \( S^* \) is an optimum combination strategy on the network system if

\[
S_1^* + S_2^* + \cdots + S_N^* = S_h^* = \frac{a_1}{a_0},
\]

\[
\frac{\partial C_0(S, P_1, P_2)}{\partial s_i} |_{S^*} = 0,
\]

\[
\frac{\partial^2 C_0(S, P_1, P_2)}{\partial s_i^2} |_{S^*} < 0,
\]

and

\[
P_1(S^*) = P_2(S^*) = 0,
\]

where \( i = 1, 2, \cdots, N, s_i^* \in \{0, 1\} \).

We choose a non-linear matrix function as incentive strategy. We get:

\[
P_1 = Q_1 \frac{dS_h}{dt},
\]

\[
P_2 = Q_2 [(s_1 - s_1^*), (s_2 - s_2^*), \cdots, (s_N - s_N^*)]^T,
\]

where \( Q_1 \) is a \( N \times 1 \) matrix and \( Q_2 \) is a \( N \times N \) matrix defined respectively via:

\[
Q_1 = (q_1, q_2, \cdots, q_N)^T,
\]

\[
Q_2 = \begin{pmatrix}
  q_{11} & q_{12} & \cdots & q_{1N} \\
  q_{21} & q_{22} & \cdots & q_{2N} \\
  \vdots & \vdots & \ddots & \vdots \\
  q_{N1} & q_{N2} & \cdots & q_{NN}
\end{pmatrix}.
\]

Substituting (16) and (17) into (10) and assume

\[
\frac{\partial c_i}{\partial s_j} |_{S^*} = 0, \quad \frac{\partial^2 c_i}{\partial s_j^2} |_{S^*} < 0 (\forall S \in E = S_1 \times S_2 \times \cdots \times S_N),
\]

we get:

\[
Q_1 = \begin{pmatrix}
  B_1 d_1 k \lambda_1^2 \sum_{j \neq 1}^{N} \lambda_j \frac{1}{2\lambda_1^2} [(1 - \pi \lambda_T)^{d_1-2}(d_1 - 1)\lambda_T - \frac{(d_1+1)\lambda_1}{k(1-\pi \lambda_T)^{d_1+1}}] \\
  B_2 d_2 k \lambda_2^2 \sum_{j \neq 2}^{N} \lambda_j \frac{1}{2\lambda_2^2} [(1 - \pi \lambda_T)^{d_2-2}(d_2 - 1)\lambda_T - \frac{(d_2+1)\lambda_2}{k(1-\pi \lambda_T)^{d_2+1}}] \\
  \vdots \\
  B_N d_N k \lambda_N^2 \sum_{j \neq N}^{N} \lambda_j \frac{1}{2\lambda_N^2} [(1 - \pi \lambda_T)^{d_N-2}(d_N - 1)\lambda_T - \frac{(d_N+1)\lambda_N}{k(1-\pi \lambda_T)^{d_N+1}}]
\end{pmatrix}.
\]
where

\( (22) \)

\[ q_i = B_i d_i k^2 \pi^2 \sum_{j \neq i}^{N} \frac{\lambda_j}{2a_i} \left[ (1 - \pi \lambda_T)^{d_i-2} (d_i-1) \lambda_T - \frac{(d_i+1) \lambda_i}{k(1-\pi \lambda_T)^{d_i+2}} \right], \]

\[ \frac{\partial w_i}{\partial s_i} = -d_i \pi w_i^{d_i-1} \sum_{t \neq i}^{N} \frac{w_t^2 \lambda_t}{k(1-\pi \lambda_T)^{d_i+t}}, \]

\[ \frac{\partial w_i}{\partial s_j} = \frac{d_i \pi \lambda_j w_i^{d_i+1}}{k(1-\pi \lambda_T)^{d_i+t}}, \]

\[ S = S^*, \text{ for } (i, j = 1, 2, ..., N, i \neq j). \]

So the function \( c_i(s_1, s_{-i}) \) is maximum at the equilibrium point \( S^* \). Formulas (16) and (17) are described as:

\( (23) \)

\[ \mathbf{P}_1 = \begin{pmatrix}
B_1 d_1 k^2 \pi^2 & \sum_{j \neq 1}^{N} \frac{\lambda_j}{2a_1} \left[ (1 - \pi \lambda_T)^{d_1-2} (d_1-1) \lambda_T - \frac{(d_1+1) \lambda_i}{k(1-\pi \lambda_T)^{d_1+2}} \right] \\
B_2 d_2 k^2 \pi^2 & \sum_{j \neq 2}^{N} \frac{\lambda_j}{2a_2} \left[ (1 - \pi \lambda_T)^{d_2-2} (d_2-1) \lambda_T - \frac{(d_2+1) \lambda_i}{k(1-\pi \lambda_T)^{d_2+2}} \right] \\
\vdots & \vdots \\
B_N d_N k^2 \pi^2 & \sum_{j \neq N}^{N} \frac{\lambda_j}{2a_N} \left[ (1 - \pi \lambda_T)^{d_N-2} (d_N-1) \lambda_T - \frac{(d_N+1) \lambda_i}{k(1-\pi \lambda_T)^{d_N+2}} \right]
\end{pmatrix} \times s_0 \left( a_0 - a_1 s_1 \right) \]

\( s^* \)

\[ \mathbf{P}_2 = \begin{pmatrix}
\frac{\partial w_1}{\partial s_1} B_1 - a_0 q_1 & \frac{\partial w_1}{\partial s_2} B_1 - a_0 q_1 & \cdots & \frac{\partial w_1}{\partial s_N} B_1 - a_0 q_1 \\
\frac{\partial w_2}{\partial s_1} B_2 - a_0 q_2 & \frac{\partial w_2}{\partial s_2} B_2 - a_0 q_2 & \cdots & \frac{\partial w_2}{\partial s_N} B_2 - a_0 q_2 \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial w_N}{\partial s_1} B_N - a_0 q_N & \frac{\partial w_N}{\partial s_2} B_N - a_0 q_N & \cdots & \frac{\partial w_N}{\partial s_N} B_N - a_0 q_N 
\end{pmatrix} - \triangle p \mathbf{T} \times \begin{pmatrix}
s_1 - s_1^* \\
s_2 - s_2^* \\
\vdots \\
s_N - s_N^*
\end{pmatrix} \]

Based on the above set-up, we have the following theorem.

**Theorem:** Strategies \( \mathbf{P}_1 \) and \( \mathbf{P}_2 \) are incentive Stackelberg strategies for the network system if \( \mathbf{P}_1 \) and \( \mathbf{P}_2 \) satisfy formulas (23) and (24), respectively.
4. An example of incentive Stackelberg strategy

Utilizing the general form for the utility function described in (5), and the well-known queuing theory results in (8), to illustrate the search for an equilibrium in the extreme points of $S$, we next investigate a two-user case with non-linear utilities.

Assume:

$$x^2 = x^2, i = 1, 2, \lambda_1 = \lambda_2, \quad A_1 = 10, B_1 = 10, A_2 = 15, B_2 = 20, d_1 = d_2 = 2.$$ 

Nash equilibrium $S^*$ of equation (5) is found to be:

$$S^* = \begin{cases} 
(1, 1), & \Delta p \leq 0.9; \\
(0, 1), & 0.9 < \Delta p \leq 1.3; \\
(0, 0), & \Delta p > 1.3.
\end{cases}$$

Suppose $a_0 = a_1 = 1$.

In this case, the state equilibrium of equation (3) satisfies

$$M^*(s^*_h) = M^*(1).$$

This result suggests the existence of an optimum price range (namely $0.9 < \Delta p \leq 1.3$) that takes advantage of the structure of the network to benefit all users according to their sensitivities to QoS.

Suppose we have two users whose applications are characterized in Table 1.

<table>
<thead>
<tr>
<th>$\Delta p = 0.7$</th>
<th>$s_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$s_1$</td>
<td>8.13, 12.50</td>
</tr>
<tr>
<td>$s_2$</td>
<td>8.17, 11.53</td>
</tr>
</tbody>
</table>

Let $\Delta p = 1.1$, Nash equilibrium of equation (5) exists in $S^* (0, 1)$. Strategy combination $S^*(0, 1)$ is Pareto optimal in the ideal state.

Let $\Delta p = 0.7$, in this case, $(1, 1)$ is a Nash equilibrium. This is the unique Nash equilibrium. However, this unique equilibrium is clearly not Pareto optimal, since $c_i(1, 1) < c_i(0, 0), (i = 1, 2)$.

We take two appropriate policies $P_1$ and $P_2$ as those in equations (23) and (24) to system (5). Let:

$$P_1 = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} s_h (1 - \frac{1}{2}s_h) = \begin{pmatrix} -0.459 \\ -0.918 \end{pmatrix} s_h (1 - \frac{1}{2}s_h),$$

$$P_2 = -\begin{pmatrix} \frac{\partial w_1}{\partial s_1} B_1 + \Delta p \lambda_1 - a_0 q_1 & \frac{\partial w_1}{\partial s_2} B_1 - a_0 q_1 \\ \frac{\partial w_2}{\partial s_1} B_2 - a_0 q_2 & \frac{\partial w_2}{\partial s_2} B_2 + \Delta p \lambda_2 - a_0 q_2 \end{pmatrix} s_h,$$

where

$$S^* = (0, 1), \quad q_i = B_i d_i k^2 \lambda_j \frac{1}{2\lambda_j} [(1 - \lambda \lambda_T) d_i - 2(d_i - 1) \lambda_T] - \frac{(d_i + 1) \lambda_j}{\lambda (1 - \lambda \lambda_T) s_i \tau_T} (i, j = 1, 2, i \neq j).$$
Typical consumers surplus combinations are shown in Table 2.

Table 2: The values of the benefit functions in incentive

<table>
<thead>
<tr>
<th>$s_2$</th>
<th>$s_1$</th>
<th>$\Delta p = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>6.37, 12.052, 7.64, 12.890</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>6.63, 12.185, 7.31, 11.464</td>
</tr>
</tbody>
</table>

The above result suggests the existence of a Nash equilibrium in $S^* (0, 1)$. This is also Pareto optimal in the ideal state.

5. Conclusion

This paper discussed the nonlinear and crossing influence incentive strategies for multi-service two-priority network systems. We demonstrated how, by adopting two appropriate incentive policies $P_1$ and $P_2$, the service provider is able to offer the needed incentives for each user to choose the service that best matches his (her) needs, the users and the network managers can get the best surplus and the system gets to an ideal and steady point which is the equilibrium point of systems (3) and (10). This conclusion is important in both theory and practice.

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