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INPUT-OUTPUT DECOUPLING FOR A CLASS OF DESCRIPTOR CONTROL SYSTEMS

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Abstract. Input-output decoupling for a class of descriptor control systems is considered in this paper. First, an algorithm is developed, and a coordinate change is introduced such that the control system can be expressed in a simple form. A feedback control law is then constructed to realize the input-output decoupling, which also leads to an impulse-free solution of the closed-loop system. Finally, a numerical example is provided to illustrate the results in this paper.

Key Words. descriptor systems; input-output decoupling; feedback control.

1. Introduction

The relations of input and output are fundamental in control systems as a realization control role. However, if relations of input and output are very complex, the relationship of some input and some output becomes obscure, and the control role of systems can not be further realized. Thus, simplify the contact of input and output of systems is an important problem in control system theory. The typical method is input-output decoupling. As for linear systems, the theory of input-output decoupling has been summarized ([1][2]), and for nonlinear systems, the research of input-output decoupling is also relatively studied, especially, relating to the vector relative degree, typical method of input-output decoupling has been developed ([3][4][5]). With regard to descriptor (or singular) systems, the results on input-output decoupling problem have not been well developed because of the particular structure of descriptor systems. Since 1970, more and more scholars from all over the world have concentrated on the research of descriptor systems, and with the development of the research, it is found that some practical systems, such as economic systems ([6]), chemical process systems ([7]) and power systems ([8]) and so on, belong to singular systems. Especially, typical constrained mechanical systems and most of the robot systems are as descriptor systems ([9]). Hence the study of input-output decoupling problem for descriptor systems is also very important and meaningful.

In this paper, we mainly discuss the problem of input-output decoupling. For a class of descriptor control systems, a new decoupling algorithm will be developed. Based on the proposed algorithm, a coordinate change is introduced such that the control system can be expressed in a simple form. A feedback control law is then constructed, which ensures that the corresponding closed-loop system has an impulse-free solution, and the input-output decoupling is realized. At the end of this paper, a numerical example is provided to illustrate the results obtained in this paper.

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2. Problems and definition

Consider a descriptor control system

$$E\dot{x} = Ax + Bu \qquad (1)$$
$$y = Cx$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the input vector, $y \in R^m$ is the output vector, E is a $n \times n$ singular matrix, A is a $n \times n$ nonsingular matrix, B is a $n \times m$ matrix, and C is a $m \times n$ matrix.

Without loss of generality, we can assume E has a rank of $r(r < n)$. So there must exist two $n \times n$ nonsingular matrix P and Q such that

$$PEQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

where I_r is a $r \times r$ unit matrix. Multiply the first equation of (1) by matrix P in the left, and let $Q^{-1}x(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$, $x_1(t) \in R^r$, $x_2(t) \in R^{n-r}$, then, System (1) can be transformed to

$$\begin{aligned} \dot{\tilde{x}}_1 &= A_{11}\tilde{x}_1 + A_{12}\tilde{x}_2 + B_1 u \\ 0 &= A_{21}\tilde{x}_1 + A_{22}\tilde{x}_2 + B_2 u \\ y &= C_1\tilde{x}_1 + C_2\tilde{x}_2 \end{aligned} \qquad (2)$$

where A_{11} is a $r \times r$ nonsingular matrix, A_{22} is a $(n-r) \times (n-r)$ matrix, A_{12}, A_{21}, B_1, B_2, C_1, and C_2 are the matrices with appropriate dimensions([10]).

For convenience, we can consider the following descriptor system with no loss of generality.

$$\begin{aligned} \dot{x} &= A_{11}x + A_{12}z + B_1 u \\ 0 &= A_{21}x + A_{22}z + B_2 u \\ y &= Cx \end{aligned} \qquad (3)$$

where $x \in R^n$ is the vector of differential variable, $z \in R^s$ is the algebraic variable, $u \in R^m$ is the output equations, A_{11} is a $n \times n$ nonsingular matrix, A_{22} is a $s \times s$ matrix, A_{12}, A_{21}, B_1, B_2, and C are the matrices with appropriate dimensions, and B_1 has a full column rank, C has a full row rank.

Consider a feedback control law with the following form

$$u = \alpha x + \gamma z + \beta \nu \qquad (4)$$

where β is nonsingular. Under the feedback law (4), System (3) becomes the following form

$$\begin{aligned} \dot{x} &= (A_{11} + B_1\alpha)x + (A_{12} + B_1\gamma)z + B_1\gamma\nu \\ 0 &= (A_{21} + B_2\alpha)x + (A_{22} + B_2\gamma)z + B_2\gamma\nu \\ y &= Cx \end{aligned} \qquad (5)$$

Definition 1. *The input-output decoupling problem of System (3) is said to be solvable, if there exist a state feedback (4) such that the closed loop systems (5) has impulse-free solution and, for each $1 \le i \le$, the output y_i is affected only by corresponding input u_i and not by y_j, if $j \neq i$.*

3. Algorithm description

In this section, we first show the following algorithm, which plays an important role in discussing the decoupling problems of System (3).

Algorithm 1.

Step 1. Set $\varphi_i^0(x) = c_i x$. Where c_i is i-row of the matrix C in Systems (3). If there exists a $n \times s$ matrix $D(x)$ such that $D(x)$ has constant rank s in a neighborhood of x_0 and satisfy

$$\begin{aligned} c_i(A_{12} + DA_{22}) &= 0 \\ c_i(B_1 + DB_2) &= 0 \end{aligned} \tag{6}$$

Denote $\varphi_i^1 = c_i(A_{11} + DA_{21})x$. Otherwise, set $r_i = 1$, and quit the algorithm.

Step 2. The previous construction can be iterated with $\varphi_i^1(x)$ instead of $\varphi_i^0(x)$, here $\varphi_i^1(x) = c_i(A_{11} + DA_{21})x$. If

$$\begin{aligned} c_i(A_{11} + DA_{21})(A_{12} + DA_{22}) &= 0 \\ c_i(A_{11} + DA_{21})(B_1 + DB_2) &= 0 \end{aligned}$$

Denote $\varphi_i^2 = c_i(A_{11} + DA_{21})^2 x$. Otherwise, set $r_i = 2$, and quit the algorithm.

Step k+1. Assume that a sequence of $\varphi_i^0(x), \cdots, \varphi_i^k(x)$ has been defined from Step 1 to Step k. For $\varphi_i^1(x) = c_i(A_{11} + DA_{21})^k(x)$, check up the equations

$$\begin{aligned} c_i(A_{11} + DA_{21})^k(A_{12} + DA_{22}) &= 0 \\ c_i(A_{11} + DA_{21})^k(B_1 + DB_2) &= 0 \end{aligned}$$

If the equations are correct, $\varphi_i^{k+1}(x) = c_i(A_{11}+DA_{21})^{k+1}x$ can be obtained. Otherwise, set $r_i = k+1$ and quit algorithm.

Perform Algorithm 1 for $i = 1, \cdots, m$ and it can produce integers $r_1, \cdots, r_m$ and the function sequence $\varphi_1^0(x), \cdots, \varphi_1^{r_1-1}(x), \cdots, \varphi_m^0(x), \cdots, \varphi_m^{r_m-1}(x)$. Set

$$\tilde{A}_{11} = A_{11} + DA_{21}, \quad \tilde{A}_{12} = A_{12} + DA_{22}, \quad \tilde{B}_1 = B_1 + DB_2$$

$$\tilde{a} = \begin{bmatrix} c_1\tilde{A}_{11}^{r_1} \\ \vdots \\ c_m\tilde{A}_{11}^{r_m} \end{bmatrix}, \quad \tilde{b} = \begin{bmatrix} c_1\tilde{A}_{11}^{r_1-1}\tilde{A}_{12} \\ \vdots \\ c_m\tilde{A}_{11}^{r_m-1}\tilde{A}_{12} \end{bmatrix}, \quad \tilde{c} = \begin{bmatrix} c_1\tilde{A}_{11}^{r_1-1}\tilde{B}_1 \\ \vdots \\ c_m\tilde{A}_{11}^{r_m-1}\tilde{B}_1 \end{bmatrix}$$

and

$$W = \begin{bmatrix} A_{22} & B_2 \\ \tilde{b} & \tilde{c} \end{bmatrix} \tag{7}$$

With regard to Algorithm 1, the following conclusion can be obtained

Lemma 1. *The integers $r_1, \cdots, r_m$ are invariant under feedback (2).*

Proof. Under the feedback (2), System (3) is with the following form

$$\begin{aligned} \dot{x}_1 &= (A_{11} + B_1\alpha)x + (A_{12} + B_1\gamma)z + B_1\gamma\nu \\ 0 &= (A_{21} + B_2\alpha)x + (A_{22} + B_2\gamma)z + B_2\gamma\nu \\ y &= Cx \end{aligned}$$

Set

$$\begin{aligned}\bar{A}_{i1} &= A_{i1} + B_i\alpha\\ \bar{A}_{i2} &= A_{i2} + B_i\gamma\\ \bar{B}_i &= B_i\beta\end{aligned}$$

for $i = 1,\ 2$. Because $\bar{\varphi}_i^0(x) = \varphi_i^0(x)$, we have

$$\begin{aligned}c_i(\bar{A}_{12} + D\bar{A}_{22}) &= c_i\left[(A_{12} + B_1\gamma) + D(A_{22} + B_2\gamma)\right]\\ &= c_i(A_{12} + DA_{22}) + c_i(B_1 + DB_2)\gamma\end{aligned}$$

$$c_i(\bar{B}_1 + D\bar{B}_2) = c_i(B_1\beta + DB_2\beta) = c_i(B_1 + DB_2)\gamma$$

Therefore, the equations

$$\begin{aligned}c_i(A_{12} + DA_{22}) = 0\\ c_i(B_1 + DB_2) = 0\end{aligned}$$

are equivalent with the equations

$$\begin{aligned}c_i(\bar{A}_{12} + D\bar{A}_{22}) = 0\\ c_i(\bar{B}_1 + D\bar{B}_2) = 0\end{aligned}$$

Notice that

$$\begin{aligned}\bar{\varphi}_i^1 &= c_i(\bar{A_{11}} + D\bar{A_{21}})x\\ &= c_i\left[(A_{11} + B_1\alpha) + D(A_{21} + B_2\alpha)\right]x\\ &= c_i\left[(A_{11} + DA_{21}) + (B_1 + DB_2)\alpha\right]x\\ &= c_i(A_{11} + DA_{21})x + c_i(B_1 + DB_2)\alpha x\\ &= c_i(A_{11} + DA_{21})x\\ &= \varphi_i^1(x)\end{aligned}$$

and we can get that $\bar{\phi}_i^k(x) = \phi_i^k(x)$. Thus, the integers $r_1, \cdots, r_m$ will be invariant under the feedback (2). □

Lemma 2. *If the matrix (7) is nonsingular, the vectors $\phi_1^0(x)$, $\cdots$, $\phi_1^{r_1-1}$, $\cdots$, ϕ_m^0, $\cdots$, $\phi_m^{r_m-1}$ are linearly independent at x_0.*

Proof. Without loss of generality, assume $r_1 \geq r_2 \geq \cdots \geq r_m$. If necessary, suppose that the order of the rows of may be changed. Consider the matrix

$$P = \begin{bmatrix}\tilde{A}_{12} & \tilde{B}_1 & \tilde{A}_{11}\tilde{A}_{12} & \tilde{A}_{11}\tilde{B}_1 & \cdots & \tilde{A}_{11}^{r_1-1}\tilde{A}_{12} & \tilde{A}_{11}^{r_1-1}\tilde{B}_1\end{bmatrix}$$

and

$$Q = \begin{bmatrix}c_1 & c_1(\tilde{A}_{11})^T & \cdots & c_1(\tilde{A}_{11}^{r_1-1})^T & \cdots & c_m & c_m(\tilde{A}_{11})^T & \cdots & c_m(\tilde{A}_{11}^{r_m-1})^T\end{bmatrix}^T$$

Then it follows from Algorithm 1 that the matrix QP exhibits a block triangular structure in which the diagonal blocks consist of rows of the matrix $\begin{bmatrix}\tilde{b} & \tilde{c}\end{bmatrix}$. According to the conditions of Lemma 2, the matrix $\begin{bmatrix}\tilde{b} & \tilde{c}\end{bmatrix}$ is of full row rank at x_0, then the row vectors of the matrix $\begin{bmatrix}\tilde{b} & \tilde{c}\end{bmatrix}$ are linearly independent at x_0. As a consequence, the matrix QP is of also full row rank at x_0. So the rows of the matrix are linearly independent at x_0, i.e. the vectors $\phi_1^0(x)$, $\cdots$, $\phi_1^{r_1-1}$, $\cdots$, ϕ_m^0, $\cdots$, $\phi_m^{r_m-1}$ are linearly independent at x_0. □

4. Main results

The section will be devoted to constructing a feedback control law of the form (2) such that the closed-loop system (5) has a unique solution with no impulse and can realize input-output decoupling. To this end, we first construct a coordinate transformation in which the system is assumed to be with a simple form, which is similar to that in [3].

Let $r = r_1 + \cdots + r_m$. Then it follows from Lemma 2 that $r \leq n$. Thus if $r < n$, there exist $(n - r)$ smooth functions such that the vectors

$$d\phi_1^0(x),\ \cdots,\ d\phi_1^{r_1-1},\ \cdots,\ d\phi_m^0,\ \cdots,\ d\phi_m^{r_m-1},\ d\eta_1(x),\ \cdots,\ d\eta_{n-r}(x)$$

are linearly independent in a neighborhood of x_0. Therefore, by inverse function theorem, the mapping

$$\Phi(x) = \left[\phi_1^0(x),\quad \cdots,\quad \phi_1^{r_1-1},\quad \cdots,\quad \phi_m^0,\quad \cdots,\quad \phi_m^{r_m-1},\quad \eta_1(x),\quad \cdots,\quad \eta_{n-r}(x)\right]$$

is a local diffeomorphism about x_0, which implies that $\Phi(x)$ constitutes a coordinate transformation.

Set

$$\xi_i^1 = \phi_i^0(x),\ \cdots,\ \xi_i^{r_i} = \phi_i^{r_i-1}(x) \tag{8}$$

for $i = 1, \cdots, m$.

$$\eta_1 = \eta_1(x),\ \cdots,\ \eta_{n-r} = \eta_{n-r}(x)$$

According to Algorithm 1, a straightforward calculation shows that System (3) in new coordinates (η, ξ) takes the form of

$$\begin{aligned} &\dot{\eta} = \hat{f}_1(\xi,\eta) + \hat{p}_1(\xi,\eta)z + \hat{g}_1(\xi,\eta)u \\ &\dot{\xi}_i^1 = \xi_i^2,\ \cdots,\ \dot{\xi}_i^{r_i-1} = \xi_i^{r_i},\ \dot{\xi}_i^{r_i} = c_i\tilde{A}_{11}^{r_i}x + c_i\tilde{A}_{11}^{r_i-1}\tilde{A}_{12}z + c_i\tilde{A}_{11}^{r_i-1}\tilde{B}_1u \\ &0 = A_{21}x + A_{22}z + B_2u \\ &y_i = \xi_i^1 \end{aligned} \tag{9}$$

where $x = \Phi^{-1}(\xi,\eta)$, $\xi = (\xi_1^1, \cdots, \xi_1^{r_i}, \cdots, \xi_m^1, \cdots, \xi_m^{r_m})^T$ and $\eta = (\eta_1, \cdots, \eta_{n-r})^T$.

Now we shall construct a feedback control law by two steps. First, choose the matrix $\gamma(x)$ such that matrix $A_{22} + B_2\gamma$ is nonsingular. Imposed the feedback law $u = \gamma z + \hat{u}$, System (9) becomes the following form.

$$\dot{\eta} = \hat{f}_1(\xi,\eta) + \left[\hat{p}_1(\xi,\eta) + \hat{g}_1(\xi,\eta)\gamma\right]z + \hat{g}_1(\xi,\eta)\hat{u} \tag{10a}$$

$$\begin{aligned} &\dot{\xi}_i^1 = \xi_i^2,\ \cdots,\ \dot{\xi}_i^{r_i-1} = \xi_i^{r_i}, \\ &\dot{\xi}_i^{r_i} = c_i\tilde{A}_{11}^{r_i}x + \left[c_i\tilde{A}_{11}^{r_i-1}\tilde{A}_{12} + c_i\tilde{A}_{11}^{r_i-1}\tilde{B}_1\gamma\right]z + c_i\tilde{A}_{11}^{r_i-1}\tilde{B}_1\hat{u} \end{aligned} \tag{10b}$$

$$0 = A_{21}x + \left[A_{22} + B_2\gamma\right]z + B_2\hat{u} \tag{10c}$$

$$y_i = \xi_i^1 \tag{10d}$$

Because the matrix $[A_{22} + B_2\gamma]$ is nonsingular, and according to (10c), z takes the following form

$$z = -\left[A_{22} + B_2\gamma\right]^{-1}\left[A_{21}x + B_2\hat{u}\right] \tag{11}$$

Substitute z in System (10) for (11), System (10) becomes following form

$$\begin{aligned} &\dot{\eta} = \bar{f}_1(\xi,\eta) + \bar{g}_1(\xi,\eta)\hat{u} \\ &\dot{\xi}_i^1 = \xi_i^2,\ \cdots,\ \dot{\xi}_i^{r_i-1} = \xi_i^{r_i},\ \dot{\xi}_i^{r_i} = \hat{a}_ix + \hat{c}_i\hat{u} \\ &y_i = \xi_i^1 \end{aligned} \tag{12}$$

where

$$\hat{a}_i = \tilde{a}_i - \left[\tilde{b}_i + \tilde{c}_i\gamma\right][A_{22} + B_2\gamma]^{-1} A_{21}$$
$$\hat{a}_i = \tilde{a}_i - \left[\tilde{b}_i + \tilde{c}_i\gamma\right][A_{22} + B_2\gamma]^{-1} A_{21}$$

for $i = 1, \cdots, m$. Because the matrix (7) is nonsingular, we know the matrix $\hat{c} = \begin{bmatrix} c_1 & \cdots & c_m \end{bmatrix}^T$ is nonsingular. In fact

$$\begin{aligned} rank \begin{bmatrix} A_{22} & B_2 \\ \tilde{b} & \tilde{c} \end{bmatrix} &= rank \left\{ \begin{bmatrix} A_{22} & B_2 \\ \tilde{b} & \tilde{c} \end{bmatrix} \begin{bmatrix} I & 0 \\ \gamma & I \end{bmatrix} \begin{bmatrix} I & (A_{22}+B_2\gamma)^{-1}B_2 \\ 0 & I \end{bmatrix} \right\} \\ &= rank \begin{bmatrix} A_{22}+B_2\gamma & 0 \\ \tilde{b}+\tilde{c}\gamma & \hat{c} \end{bmatrix} \end{aligned}$$

From the non-singularity of the matrix (7) and $A_{22} + B_2\gamma$, we get that the matrix $\hat{c}$ is nonsingular. Hence, let

$$\hat{a}_i x + \hat{c}_i \hat{u} = \nu_i \qquad i = 1, \cdots, m$$

or

$$\hat{u} = -\hat{c}^{-1}(\hat{a}x - \nu) \tag{13}$$

Under the feedback (13), Systems (12) becomes

$$\begin{aligned} &\dot{\eta} = f(\xi, \eta) + g(\xi, \eta)\nu \\ &\dot{\xi}_i^1 = \xi_i^2, \ \cdots, \ \dot{\xi}_i^{r_i - 1} = \xi_i^{r_i}, \ \dot{\xi}_i^{r_i} = \nu_i \\ &y_i = \xi_i^1 \end{aligned} \tag{14}$$

It is easily seen that System (14) has a unique solution without impulses and has realized input-output decoupling. Moreover, the feedback control law imposed on the system is given by

$$u = -\hat{c}^{-1}\hat{a}x + \gamma z + \hat{c}^{-1}\nu$$

As a consequence, we can get the following conclusion.

Theorem 1. *For the descriptor system (3), if there exists a matrix such that the matrix (7) is nonsingular, then the input-output decoupling problem of the system (3) is solvable, i.e. there exist a state feedback of the form (2) such that the corresponding closed loop systems has impulses free solution, and can realize input-output decoupling.*

5. An example

In the section a numerical example is provided to illustrate the results proposed in this paper. Consider the following descriptor system.

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 2 & 0 & 0 & 2 & 0 \\ 1 & 1 & 0 & 0 & -1 \\ 2 & 0 & 3 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 2 & 0 \\ 0 & 1 \\ 1 & 0 \\ -1 & 1 \\ 1 & 0 \end{bmatrix} z + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 2 & 1 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} u \\ 0 &= \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 2 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} z + \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} u \\ y_1 &= x_1 - x_5, \quad y_2 = x_1 - x_2 + x_4 \end{aligned} \tag{15}$$

First, let

$$\varphi_1^0(x) = y_1 = x_1 - x_5$$
$$\varphi_2^0(x) = y_2 = x_1 - x_2 + x_4$$

According to Algorithm 1 in section 3, we may select a matrix

$$D = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}^T$$

By straightforward calculating, we know that

$$c_1(A_{12} + DA_{22}) = 0$$
$$c_1(B_1 + DB_2) = 0$$
$$c_2(A_{12} + DA_{22}) = 0$$
$$c_2(B_1 + DB_2) = 0$$

Therefore, we can obtain

$$c_1(A_{12} + DA_{22}) = 0$$
$$c_1(B_1 + DB_2) = 0$$

However, a simple calculation shows that

$$c_1(A_{11} + DA_{21})(A_{12} + DA_{22}) \neq 0$$
$$c_1(A_{11} + DA_{21})(B_1 + DB_2) \neq 0$$
$$c_2(A_{11} + DA_{21})(A_{12} + DA_{22}) \neq 0$$
$$c_2(A_{11} + DA_{21})(B_1 + DB_2) \neq 0$$

Hence, $r_1 = r_2 = 2$.

Because $n = 5$, we need to increase $\eta_1 = x_3$. Thus the coordinate transformation $(\xi, \eta) = \Phi(x)$ is as follows

$$\begin{aligned} &\xi_1^1 = x_1 - x_5, \quad \xi_1^2 = x_2 \\ &\xi_1^2 = x_1 - x_2 + x, \quad \xi_2^2 = x_5 \\ &\eta_1 = x_3 \end{aligned} \tag{16}$$

According to Algorithm 1, a simple calculation shows that system (15) in new coordinates (16) takes the form of

$$\dot{\eta}_1^1 = 2x_1 + 3x_3 + x_5 + z_1 + 2u_1 + u_2 \tag{17a}$$

$$\begin{aligned} &\dot{\xi}_1^2 = \xi_1^2,\ \dot{\xi}_1^2 = x_1 + x_2 - x_5 + z_2 + u_1, \\ &\dot{\xi}_2^2 = \xi_2^2,\ \dot{\xi}_2^2 = x_1 - x_2 + x_4 + z_1 + u_2 \end{aligned} \tag{17b}$$

$$0 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 2 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} z + \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} u \tag{17c}$$

$$y_1 = \xi_1^1,\ y_2 = \xi_2^1 \tag{17d}$$

where $x = \Phi^{-1}(\xi, \eta)$. Let

$$\begin{aligned} x_1 + x_2 - x_5 + z_2 + u_1 = \nu_1 \\ x_1 - x_2 + x_4 + z_1 + u_2 = \nu_2 \end{aligned} \tag{18}$$

It is obvious that the matrix

$$W = \begin{bmatrix} A_{22} & B_2 \\ \tilde{b} & \tilde{c} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

is nonsingular. Hence, from equations (17c) and (18), we can further obtain

$$\begin{aligned} z_1 &= -x_1 - x_4 \\ z_2 &= x_1 + 2x_4 + x_5 + \nu_1 + \nu_2 \\ u_1 &= -2x_1 - x_2 - 2x_4 - \nu_2 \\ u_2 &= x_2 + \nu_2 \end{aligned} \tag{19}$$

Finally, by imposing the feedback (18) on System (16) and noticing (19), we can get that

$$\begin{aligned} \dot{\eta}_1^1 &= -3x_1 - x_2 + 3x_3 - 5x_4 + x_5 - \nu_2 \\ \dot{\xi}_1^2 &= \xi_1^2,\ \dot{\xi}_1^2 = \nu_1, \\ \dot{\xi}_2^2 &= \xi_2^2,\ \dot{\xi}_2^2 = \nu_2 \\ y_1 &= \xi_1^1,\ y_2 = \xi_2^1 \end{aligned} \tag{20}$$

It can be known that Systems (20) has a unique solution with no impulse. The input-output decoupling has been realized and the feedback control law imposed on the system is with the following form

$$u = \begin{bmatrix} -x_1 - x_2 + x_5 \\ -x_1 + x_2 - x_4 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} z + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \nu$$

6. Conclusions

Mainly discussed in this paper, the decoupling problems of the descriptor systems can be solved by means of a new decoupling algorithm. A new coordinate transformation (in which the system assumed to be with a simple form) can be constructed based on this algorithm, and the feedback control law, which ensures the corresponding closed-loop system may realize input-output decoupling, can be obtained.

It should be noted that results developed in this paper will provide a new method in which the decoupling problems of descriptor systems can be considered, and it can be used to solve several other control problems, for instance, stabilization, disturbance decoupling, output tracking, etc.

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