NON-FRAGILE PASSIVE CONTROL FOR UNCERTAIN SINGULAR TIME-DELAY SYSTEMS

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Abstract. This paper concerns the problem of non-fragile passive control for uncertain singular time-delay systems with time-invariant norm-bounded parameter uncertainty. The state feedback gains are with norm-bounded controller uncertainties. The problem of designing memoryless state feedback for both cases with additive and multiplicative controller uncertainties are addressed. Sufficient condition, which guarantees the prescribed singular time-delay system is admissible and strictly passive, is given using linear matrix inequality (LMI) and generalized constraints. Furthermore, sufficient conditions for the solvability of the non-fragile passive control problem, for the cases with additive and multiplicative controller uncertainties, are presented respectively. The controllers can be obtained via solving linear matrix inequality (LMI) and generalized constraints. Finally, a numeric example is given to illustrate the validity of the method.

Key Words. Singular-time delay systems, uncertain systems, controller fragility, admissibility, strictly passive control, linear matrix inequality (LMI).

1. Introduction

Positive realness plays an important role in control system theory. The positive real control problem has been a topic of recurring interest over the past years. The objective of the positive real control problem is to design a controller such that the resulting closed-loop system is stable and the closed-loop transfer function is positive real. We have obtained many results on the positive real control problem for normal linear systems [1-5]. Because of the fact that singular systems better describe physical systems than regular ones, control of singular systems has been extensively studied in the past years. The positive realness for singular system is studied in [6], but the positive real control problem for singular time-delay systems is more complex and still open if considering the time-delay. For time-invariant linear systems, strict passivity is equivalent to generalized strictly positive realness, and there are contacts between strict passivity and strictly positive realness for singular systems [7]. The notion of passivity is proposed for discrete-time singular systems [7], and the robust passive control problem for continuous-time singular system with time-delay is studied in [8]. In this note, we retell this problem via a new method and consider parameter uncertainties. In the study of passive control for uncertain systems, we mainly consider the controllers that are robust with respect to system uncertainties [9]. But controller uncertainties exist in many cases,
which can be caused by many reasons, such as finite word length in digital systems, the imprecision inherent in analog systems. This has motivated the study of non-fragile control problems, which are concerned with how to design controllers that are non-fragile in the sense that satisfactory performance level of the closed-loop system can be preserved in face of controller uncertainties. The non-fragile positive real control problem for uncertain neutral delay systems is studied in [10]. However, the non-fragile passive control problem is still open, which motivates this study. This paper considers the non-fragile passive control problem for uncertain singular time-delay systems. The parameter uncertainties under consideration are supposed to be time-invariant norm-bounded. The problem addressed is twofold: the first is to develop a sufficient condition which guarantees the prescribed singular time-delay system is admissible and strictly passive, the second is to give the sufficient conditions for the solvability of the non-fragile passive control problem respectively, for the cases with additive and multiplicative controller uncertainties. The controller can be obtained via solving generalized constrains and certain LMIs, and the design is easily realized by using the soft MATLAB. Finally, a numerical example is provided to demonstrate the validity of this method.

Notation: Throughout this paper, for real symmetric matrices $X$ and $Y$, the notation $X \geq Y$ (respectively $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). $I$ is the identity matrix with appropriate dimension. The superscripts "$\top$" and "$-1$" denote the transpose and the inverse. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

Consider a class of uncertain singular time-delay systems described by

\begin{align}
(1) \quad & (\Sigma) \quad E \dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - \tau) + (B_1 + \Delta B_1)u(t) + B\omega(t), \\
(2) \quad & z(t) = Cx(t) + D\omega(t), \\
(3) \quad & x(t) = \phi(t), \ t \in [-\tau, 0],
\end{align}

where $x(t) \in \mathbb{R}^n$ is the state, $\omega(t) \in \mathbb{R}^p$ is the exogenous input, $u(t) \in \mathbb{R}^m$ is the control input, $z(t) \in \mathbb{R}^p$ is the controlled output. $E \in \mathbb{R}^{n \times n}$, $A$, $A_d$, $B$, $B_1$, $C$, $D$ are known constant matrices. We shall assume that rank$E = r < n$, $\tau \geq 0$ is a constant time-delay, $\phi(t)$ is a compatible vector valued continuous function. $\Delta A$, $\Delta A_d$, $\Delta B_1$ are time-invariant parameter uncertainties, and are assumed to be of the form

\begin{align}
(4) \quad & [ \Delta A \ \Delta A_d \ \Delta B_1 ] = MF \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix},
\end{align}

where $M$, $N_1$, $N_2$ and $N_3$ are constant matrices, and $F \in \mathbb{R}^{p \times k}$ is the uncertain matrix satisfying

\begin{align}
(5) \quad & F^TF \leq I.
\end{align}

Distinct from system $(\Sigma)$ is the nominal system

\begin{align}
(6) \quad & (\Sigma_N) \quad E \dot{x}(t) = Ax(t) + A_d x(t - \tau) + B\omega(t), \\
(7) \quad & z(t) = C x(t) + D\omega(t), \\
(8) \quad & x(t) = \phi(t), \ t \in [-\tau, 0].
\end{align}
The purpose of this section is to give a condition for admissibility and strictly passivity for singular time-delay systems described by $(\Sigma_N)$. In view of this, we introduce the following singular time-delay system

\[ E \dot{x}(t) = Ax(t) + A_d x(t - \tau). \]

**Definition 1:** The singular delay system (9) is said to be admissible if the system (9) is regular, impulse free and stable.

**Remark 1:** The notions of the singular delay system (9) to be regular, impulse free and stable are given in [11].

**Definition 2:** The singular delay system $(\Sigma_N)$ is said to be strictly passive if there exists a function $V(x(t)) \geq 0$ such that the passive inequality

\[ V(x(t)) < z^\top(t) \omega(t), \forall t \geq 0 \]

holds for all $\omega(t)$.

**Lemma 1:** The singular delay system (9) is admissible if there exist symmetric matrix $Q > 0$ and invertible matrix $P$ such that the following inequalities hold

\[ E^\top P = P^\top E \geq 0, \]

\[ A^\top P + P^\top A + P^\top A_d Q^{-1} A_d^\top P + Q < 0. \]

**Proof:** According to Theorem 1 in [11], this Lemma can be obtained by simple matrix computation.

**Theorem 1:** The singular delay system $(\Sigma_N)$ is admissible and strictly passive if $D + D^\top > 0$ and there exist matrix $Q > 0$ and invertible matrix $P$ satisfying

\[ E^\top P = P^\top E \geq 0, \]

\[ \begin{bmatrix} A^\top P + P^\top A + Q & P^\top A_d & P^\top B - C^\top \\ A_d^\top P & -Q & 0 \\ B^\top P - C & 0 & -(D + D^\top) \end{bmatrix} < 0. \]

**Proof:** By Schur complements[12], it follows from (13) that

\[ A^\top P + P^\top A + P^\top A_d Q^{-1} A_d^\top P + Q < 0. \]

Therefore, by Lemma 1, we conclude the system $(\Sigma_N)$ is admissible. Let

\[ V_1(x(t)) = x^\top(t) E^\top P x(t) + \int_{t-\tau}^t x^\top(s) Q x(s) ds. \]

Then $V_1(x(t)) \geq 0$. Differentiating (14), we obtain

\[ V_1'(x(t)) - z^\top(t) \omega(t) - \omega^\top(t) z(t) = \begin{bmatrix} x^\top(t) & x^\top(t - \tau) & \omega^\top(t) \end{bmatrix} \begin{bmatrix} A^\top P + P^\top A + Q & P^\top A_d & P^\top B - C^\top \\ A_d^\top P & -Q & 0 \\ B^\top P - C & 0 & -(D + D^\top) \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau) \\ \omega(t) \end{bmatrix}. \]

Then it follows from (13) that

\[ V_1'(x(t)) < 2z^\top(t) \omega(t). \]

Put $V(x(t)) = \frac{1}{2} V_1(x(t))$, by using Definition 2, we conclude that the system $(\Sigma_N)$ is strictly passive.
Remark 2: When $E$ equals to $I$, the conditions given by Theorem 1 are the sufficient conditions for linear delay systems to be stable and strictly passive from [4].

2. Passive Control Design

For system $(\Sigma)$, we now consider a state feedback controller in the following form:

$$u(t) = (K + \Delta K)x(t),$$

where $K \in \mathbb{R}^{m \times n}$ is the controller gain to be designed, and $\Delta K$ is the controller gain perturbation.

In this paper, the following two classes of controller gain perturbations will be considered:

(I) $\Delta K$ is with the norm-bounded additive form

$$\Delta K = \Delta_1 = H_1 F_1 E_1,$$

where $H_1$ and $E_1$ are known matrices, and $F_1$ is an unknown matrix satisfying

$$F_1^T F_1 \leq I.$$ (17)

(II) $\Delta K$ is with the norm-bounded multiplicative form

$$\Delta K = \Delta_2 = H_2 F_2 E_2 K,$$

where $H_2$ and $E_2$ are known matrices, and $F_2$ is an unknown matrix satisfying

$$F_2^T F_2 \leq I.$$ (19)

The non-fragile passive control problem to be addressed in this paper is the design of a state feedback controller in the form of (15) with the gain perturbations satisfying (16) and (17), or (18) and (19), respectively, such that the resulting closed-loop system is admissible and strictly passive for all uncertainties.

Lemma 2[13]: Let $T, \Phi, \Psi, \Omega$ and $F$ be real matrices of appropriate dimensions such that $\Omega > 0$ and $F^T F \leq I$. Then we have the following:

1. For scalar $\varepsilon > 0$,

$$\Phi F \Psi + (\Phi F \Psi)^T \leq \varepsilon^{-1} \Phi \Phi^T + \varepsilon \Psi^T \Psi.$$ (21)

2. For any scalar $\varepsilon > 0$ such that $\Omega - \varepsilon \Phi \Phi^T > 0$,

$$(T + \Phi F \Psi)^T \Omega^{-1} (T + \Phi F \Psi) \leq T^T (\Omega - \varepsilon \Phi \Phi^T)^{-1} T + \varepsilon^{-1} \Psi^T \Psi.$$ (22)

Now we are in a position to present the solvability condition for the non-fragile positive real control problem for uncertain singular time-delay system $(\Sigma)$ with the controller perturbation $\Delta_1$ in (16)–(17).

Theorem 2: Consider the uncertain singular time-delay system $(\Sigma)$ and the controller perturbation $\Delta_1$ in (16) and (17). Then the non-fragile passive control problem is solvable if there exist invertible matrix $X$, symmetric matrix $Q > 0$, a matrix $Y$ and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and $\varepsilon_3 > 0$ such that the following inequalities hold

$$X^T E^T = EX \geq 0.$$ (20)
\[ J_1 = AX + X^T A^T + B_1 Y + Y^T B_1^T + (\varepsilon_1 + \varepsilon_3) M M^T, J_2 = \begin{bmatrix} -\varepsilon_2 I & \varepsilon_2 H_1^T N_2^T \\ \varepsilon_2 N_3 H_1 & -\varepsilon_3 I \end{bmatrix}, \]

\[ H = \begin{bmatrix} \varepsilon_2 B_1 H_1 & 0 \end{bmatrix}. \]

In this case, a state feedback controller chosen by

\[ u(t) = K x(t), \quad K = Y X^{-1} \]

will be such that, for all parameter uncertainties \( \Delta A, \Delta A_d, \Delta B_1 \) and the controller gain perturbations in (16) and (17), the resulting closed-loop system is admissible and strictly passive.

**Proof:** The closed-loop system of uncertain singular system \( (\Sigma) \) under the state feedback controller in (15) with \( K \) being given in (22) is

\[ (\Sigma_c) \dot{x} (t) = A_c x(t) + A_{dc} x(t - \tau) + B \omega(t), \]

\[ z(t) = C x(t) + D \omega(t), \]

\[ x(t) = \phi(t), \quad t \in [-\tau, 0], \]

where

\[ A_c = A_K + \Delta A_K + (B_1 + \Delta B_1) \Delta K, \quad A_{dc} = A_d + \Delta A_d, \quad A_K = A + B_1 K, \]

\[ \Delta A_K = \Delta A + \Delta B_1 K. \]

It is easy to know that

\[ \Delta A_K = MF \tilde{N}_1, \quad \tilde{N}_1 = N_1 + N_3 K. \]

Put \( P = X^{-1} \). Then, pre- and post-multiplying (20) by \( X^{-T} \) and \( X^{-1} \), we get

\[ E^T P = P^T E \geq 0. \]

Pre- and post-multiplying (21) by \( \text{diag}(X^{-T}, I, I, I, I, I) \) and \( \text{diag}(X^{-1}, I, I, I, I, I) \), respectively, and applying the Schur complement formula, we get

\[ \begin{bmatrix} \Lambda & P^T A_d & P^T B - C^T \\ A_d^T P & -Q & 0 \\ B^T P - C & 0 & -(D + D^T) \end{bmatrix} + \varepsilon_1^{-1} \begin{bmatrix} \tilde{N}_1^T \\ N_2^T \end{bmatrix} \begin{bmatrix} N_3 H_1 \end{bmatrix} < 0, \]

where

\[ \varepsilon_1^{-1} I - \varepsilon_3^{-1} H_1^T N_3^T N_3 H_1 > 0, \]

\[ \Lambda = P^T A_K + A_K^T P + \Pi + \varepsilon_2^{-1} E_1^T E_1, \]

\[ \Pi = (\varepsilon_1 + \varepsilon_3) P^T M M^T P + P^T B_1 H_1 (\varepsilon_2^{-1} I - \varepsilon_3^{-1} H_1^T N_3^T N_3 H_1) - H_1^T B_1^T P. \]

Noting (28) and using Lemma 2, we have

\[ \begin{bmatrix} \Gamma \\ \Delta A_d \end{bmatrix} \preceq \varepsilon_1 \begin{bmatrix} P^T M \\ 0 \end{bmatrix} \begin{bmatrix} P^T M \\ 0 \end{bmatrix}^T + \varepsilon_1^{-1} \begin{bmatrix} \tilde{N}_1^T \\ N_2^T \end{bmatrix} \begin{bmatrix} N_3 H_1 \\ N_3 H_1 \end{bmatrix} \]
3. Illustrative Example

In this case, a state feedback controller chosen by

$$J = \Delta A_K^TP + P^T\Delta A_K.$$ (31)

$$\Xi = \varepsilon_2P^T(B_1 + \Delta B_1)H_1H_1^T(B_1 + \Delta B_1)^TP \leq \Xi + \varepsilon_2^{-1}E_1^TE_1,$$

and

$$\varepsilon_2(B_1 + \Delta B_1)H_1H_1^T(B_1 + \Delta B_1)^TP \leq BH_1(\varepsilon_2^{-1}I - \varepsilon_3^{-1}H_1^TN_3^TN_3H_1)^{-1}H_1^TB_1^T + \varepsilon_3MM^T.$$ (32)

Then, by (26), (27)-(32), we get

$$\Gamma = \Delta$$

Finally, according to (33) and applying Theorem 1, we have that the closed-loop system $(\Sigma_c)$ is admissible and passive.

In the following, we will give the solvability condition for non-fragile passive control problem for uncertain singular time-delay system $(\Sigma)$ with the controller perturbation $\Delta_2$ in (18)–(19).

**Theorem 3:** Consider the uncertain singular time-delay system $(\Sigma)$ and the controller perturbation $\Delta_2$ in (18) and (19). Then the non-fragile passive control problem is solvable if there exist invertible matrix $X$, symmetric matrix $Q > 0$, a matrix $Y$ and scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ and $\varepsilon_3 > 0$ such that the following inequalities hold

$$X^TE^T = EX \geq 0,$$

and

$$\begin{bmatrix}
J_1 & A_d & B - X^TC^T & (N_1X + N_3Y)^T & Y^TE_2^T & \tilde{H}
\end{bmatrix} < 0,
$$

where $J_1$ is given in Theorem 1, and

$$\tilde{J}_2 = \begin{bmatrix}
-\varepsilon_2I & \varepsilon_2H_2^TN_3^T & -\varepsilon_3I \\
\varepsilon_2^2N_3H_2 & -\varepsilon_3I
\end{bmatrix}, \tilde{H} = \begin{bmatrix}
\varepsilon_2B_1H_2 & 0
\end{bmatrix}.$$

In this case, a state feedback controller chosen by

$$u(t) = Kx(t), K = YX^{-1}$$

will be such that, for all parameter uncertainties $\Delta A$, $\Delta A_d$, $\Delta B_1$ and the controller gain perturbations in (18) and (19), the resulting closed-loop system is admissible and strictly passive.

**Proof:** The proof is similar to the proof of Theorem 2, and thus is omitted.

3. Illustrative Example

Consider the uncertain singular delay system $(\Sigma)$ with parameters as follows:

$$E = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}, A = \begin{bmatrix}
0.1 & 1 & 0.1 \\
0.1 & 0.3 & 0.1 \\
0.5 & 0.2 & 0.1
\end{bmatrix}, A_d = \begin{bmatrix}
0.1 & 0 & 0.2 \\
0 & 0.1 & 0 \\
0 & 0.1 & -0.2
\end{bmatrix},$$

$$B_1 = \begin{bmatrix}
0.1 & 0 \\
0 & 1 \\
-1 & 1
\end{bmatrix}, B = \begin{bmatrix}
0.1 & 0.2 \\
0 & 0.1 \\
0.1 & 0
\end{bmatrix}, C = \begin{bmatrix}
0.1 & 0 & -0.1 \\
0.2 & 0.5 & 0.1
\end{bmatrix}.$$
\[ D = \begin{bmatrix} 1 & 0.1 \\ 0.5 & 1 \end{bmatrix}. \]

The uncertain matrices are assumed to satisfy (4) and (5) with
\[ M = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.2 \end{bmatrix}, N_1 = \begin{bmatrix} 0.1 & 0 & 0.1 \end{bmatrix}, N_2 = \begin{bmatrix} 0.2 & 0 & -0.1 \end{bmatrix}, N_3 = \begin{bmatrix} 0 & 0.1 \end{bmatrix}. \]

The purpose of this example is to design a state feedback controller such that the resultant closed-loop system is admissible and strictly passive for all uncertainties. Suppose the actual controller is with additive perturbations in the form of (16) and (17) with parameters as
\[ H_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, E_1 = \begin{bmatrix} 0.1 & 0 & 0.3 \end{bmatrix}. \]

Using Matlab LMI control Toolbox to solve the LMI (21) satisfying the generalized constrains (20), we obtain the solution as follows:
\[ X = \begin{bmatrix} 2.9242 & -1.5164 & 0 \\ -1.5164 & 0.8520 & 0 \\ -0.6690 & 0.4991 & -0.1295 \end{bmatrix}, Y = \begin{bmatrix} 0.8714 & -1.6067 & 1.7085 \\ -0.4568 & -1.4742 & 0.3093 \end{bmatrix}, \]
\[ Q = \begin{bmatrix} 2.6177 & 0 & 0 \\ 0 & 2.6177 & 0 \\ 0 & 0 & 2.6177 \end{bmatrix}, \]
\[ \varepsilon_1 = 2.6147, \varepsilon_2 = 2.5913, \varepsilon_3 = 2.6147. \]

Therefore, by Theorem 2, we know that the non-fragile passive control problem is solvable. A desired state feedback control controller can be obtained as
\[ u(t) = \begin{bmatrix} 4.0115 & 12.9824 & -13.1938 \\ -11.3396 & -20.5123 & -2.3888 \end{bmatrix} x(t). \]

4. Conclusion

In this paper, the problem of non-fragile passive control problem for uncertain singular time-delay systems with time-invariant norm-bounded parameter uncertainties has been studied. Admissibility and passivity conditions for uncertain singular systems have been established for the cases with additive and multiplicative controller uncertainties, respectively. A numeric example is presented to illustrate the effectiveness of the proposed approach. The positive real control problem for singular delay system is going to be studied in the future.

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