INTEGRITY CONTROLLER DESIGN
FOR DESCRIPTOR SYSTEMS

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Abstract. This short paper discusses fault-tolerant control problems for linear time-invariant descriptor systems. Firstly, a sufficient and necessary condition that descriptor systems are admissible (i.e. regular, stable and impulse-free) is obtained. Secondly, some sufficient and necessary conditions are gotten so that controllers exist in descriptor systems such that the resultant closed-loop systems are admissible. Thirdly, using generalized Lyapunov matrix equations, we gain the conditions when there are integrity controllers for descriptor systems, and give the design method. Lastly a numerical example is used to illustrate the main results.

Key Words. generalized Lyapunov equation, descriptor systems, integrity, actuator failures, admissible.

1. Introduction

The problem of stabilization of standard state-space systems is of both practical and theoretical importance and has attracted the attention of many researchers. A number of significant results on this issue have been reported and different approaches have been proposed in earlier literatures. Interested readers are referred, for instance, to see [1], [2]. The systems may become unstable when the feedback signals are switched off by actuator failures. We say that the systems possess integrity if those still keep stable in the presence of such failures. In the last decades, a great deal of attention has been paid to the integrity controller of the systems. A sequential design procedure is presented to obtain an optimal state feedback such that the systems possess integrity and good response ([3], [4]). A design method possessing integrity based on Riccati-type equation is given in [3], where they propose a synthesis method for the systems with both good dynamic feature and integrity against actuator failures. The integrity is also investigated by utilizing a positive semidefinite solution of the Lyapunov equation ([4]). This method has certain merits in using a linear equation, but there is no direct guide for control system design.

Very recently, the related topic was extended into descriptor systems ([5], [6], [7]). Studies of descriptor systems began at the end of 1970’s ([8]). Descriptor systems capture the dynamic behavior of many natural phenomena, and have applications in many fields, such as network theory, robotics, economics, biological systems, and so on. The stability and stabilization for the systems are of importance. It is discussed (for example, in [9], [10], [11]), and the results on the topics are fewer than...
those on normal systems. Generally speaking, there are two main kinds of stabilization problems for descriptor systems. One is to determine state feedback controllers such that the closed-loop descriptor systems are admissible ([10],[11]). The other is to design state feedback controllers such that the closed-loop descriptor systems are regular and stable ([12]). On the other hand, many of the existing techniques for descriptor systems are based on the assumption that the systems under consideration are regular. Descriptor systems have a great capacity for systems modeling since they can preserve the structure of physical systems and include nondynamic constraints and impulsive behavior.

Descriptor systems integrity is one fault-tolerant property of descriptor systems ([5],[6]). Few articles so far have discussed the problem of fault-tolerant control for descriptor systems. It is unavoidable that descriptor systems may suffer from actuator failures in applications due to unexpected factors in operating conditions. It is necessary to consider fault-tolerant control problems when the practical control systems are designed. Duan et al. investigated robust fault detection for descriptor systems with unknown disturbance ([7]). Chen et al studied robust quadratic stability integrity for descriptor systems. There are still a lot of topics about fault-tolerant control, such as reliability and simultaneous stabilization, open to us.

The systems may become unstable when the feedback signals are broken by actuator failures. We say that the descriptor systems have integrity if these systems remain admissible in the presence of such failures. The purpose of this brief paper is to research the integrity property for descriptor systems and design controllers to make the closed-loop descriptor systems integrable.

The paper is organized as follows: in section 2, the state-space model of the systems and the problem statement are demonstrated; in section 3, the integrity controllers are designed for the systems; in section 4, a numerical example is given to illustrate the result; a conclusion is made in the last section.

2. Systems description and problem statement

Consider linear time-invariant descriptor systems of the form:

\[
E \frac{dx(t)}{dt} = Ax(t) + Bu(t),
\]

where \(x(t) \in \mathbb{R}^n\) is the state, \(u(t) \in \mathbb{R}^m\) is the input; \(E, A\) and \(B\) are the matrices of appropriate sizes, respectively. Usually, \(\text{rank}(E) < n\). The state feedback is

\[
u(t) = -Fx(t),
\]

where \(F\) is a feedback matrix with appropriate size. Then the closed-loop descriptor systems are

\[
E \frac{dx(t)}{dt} = (A - BF)x(t).
\]

We use matrix \(L\) to describe actuator failures, and define matrix \(L\) as:

\[
L = \text{diag}(l_1, l_2, \ldots, l_m), l_i \in [0, 1].
\]

System actuators are in normal operation when \(l_i\) is equal to 1, \(l_i \in (0, 1)\) denotes system actuator failures in different degrees, \(l_i = 0\) denotes that system actuators are completely invalid. Then the resultant closed-loop descriptor systems become:
Remark 1. The problem formulation given above (3)–(4) depicting actuator failures is different from that in [13]. Yang studied the problem of standard state-space systems fault-tolerant control, and believed that \(i_i\) is 1 or 0. But we believe that \(i_i\) can continue to change from 0 to 1, and the problem model being debated is also different.

The problem is to design a controller such that the resultant closed-loop descriptor systems (4) are still admissible, when actuator failures happen, i.e. descriptor systems (1) have integrity.

3. Main results

Lemma 1. ([9]) If all the matrices \(J, F\) and \((J + KFM)\) are nonsingular, then \((J + KFM)^{-1} = J^{-1} - J^{-1}K(F^{-1} + MJ^{-1}K)^{-1}MJ^{-1}\), holds.

Construct a generalized Lyapunov function

\[
 v(Ex(t)) = x^T(t)E^TQEx(t),
\]

where \(Q = Q^T > 0\), with appropriate size. Obviously, \(v(Ex(t)) > 0\), if \(Ex(t) \neq 0\).

From (5), we get a Lyapunov equation:

\[
 E^TQ + A^TQE = -E^TPE,
\]

where \(P = P^T \in R^{n \times n}\). Equation (6) is called a generalized Lyapunov equation of descriptor systems (1).

Consider systems (1) and corresponding generalized Lyapunov equation (6), we can obtain the following theorem:

Theorem 1. Descriptor systems (1) are admissible if and only if for arbitrary positive definite symmetric matrix \(P\), generalized Lyapunov equation (6) has positive definite solution \(Q\) and

\[
 \text{rank}[E^T A^T] = n.
\]

Proof: Sufficiency. If descriptor systems (1) are not regular, there is a constant \(w \in R\) and a nonzero vector \(x \in R^n\) satisfying the following equation:

\[
 jwEx = Ax.
\]

So vector \(Ex \in R^n\) is nonzero, otherwise, we obtain \(Ax = 0\), i.e.

\[
 \text{rank}[E^T A^T] < n.
\]

It is in contradiction with \(\text{rank}[E^T A^T] = n\).

In (6), premultiplication by \(x^*\) and postmultiplication by \(x\), respectively, we obtain

\[
 0 = jwx^*E^TQEx - jwx^*E^TQEx = -x^*E^TPEx.
\]

Since \(Ex\) is a nonzero vector, it is impossible that (7) holds. Therefore, descriptor systems (1) are regular.
Without loss of generality, it is assumed that the matrices $E$ and $A$ in (1) have the following forms

$$E = \begin{bmatrix} I_r & 0 \\ 0 & N \end{bmatrix}, \quad A = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix},$$

where $I_r \in \mathbb{R}^{r \times r}$ and $I_{n-r} \in \mathbb{R}^{(n-r) \times (n-r)}$ are identity matrices, respectively, $N$ is nilpotent matrix with nilpotent index $h$, i.e. $N^{h-1} \neq 0$, $N^h = 0$. Now let $P$ and $Q$ be partitioned conformably with $E$ and $A$ as

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}, \quad Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_1^T & Q_3 \end{bmatrix}.$$  

Substituting equation (8) and (9) into generalized Lyapunov equation (6), we get

$$H = \begin{bmatrix} H_1 & H_2 \\ H_2^T & H_3 \end{bmatrix} = \begin{bmatrix} -P_1 & -P_2N \\ -N^T P_2^T & -N^T P_3 N \end{bmatrix},$$

where

$$H_1 = Q_1 A_1 + A_1^T Q_1,$$
$$H_2 = Q_2 + A_1^T Q_2 N,$$
$$H_3 = N^T Q_3 + Q_3 N.$$

If $N$ is nonzero, it follows from equation (10) that

$$N^T Q_3 + Q_3 N = -N^T P_3 N.$$  

In (11), premultiplication by $(N^{h-1})^T$ and postmultiplication by $N^{h-2}$, respectively, we have

$$(N^{h-1})^T Q_3 N^{h-2} + (N^{h-1})^T Q_3 N^{h-1} = -(N^h)^T P_3 N^{h-1}.$$  

Therefore, we obtain

$$(N^{h-1})^T Q_3 N^{h-1} = 0.$$  

According to $Q > 0$, we have $Q_3 > 0$, thus, $(N^{h-1})^T Q_3 N^{h-1} \neq 0$. It is in contradiction with (12). So $N$ is a zero matrix, i.e. descriptor systems (1) are impulse-free.

To show the stability of descriptor systems (1), we can get from (10) that

$$Q_1 A_1 + A_1^T Q_1 = -P_1,$$

where $P_1$ and $Q_1$ are positive definite matrices. By employing well known Lyapunov stability theory, it is concluded that descriptor systems (1) are asymptotically stable.

**Necessity.** Because descriptor systems (1) are regular and impulse-free, without loss of generality, we can assume that matrices $E$ and $A$ in (1) have the following forms

$$E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix}.$$ 

According to equation (10), we can get the following equation:

$$\begin{bmatrix} Q_1 A_1 + A_1^T Q_1 & Q_2 \\ Q_1^T & Q_3 \end{bmatrix} = \begin{bmatrix} -P_1 & 0 \\ 0 & 0 \end{bmatrix},$$

where $A_1 \in \mathbb{R}^{r \times r}$ is a stable matrix. Since $A_1$ is stable and $P_1 > 0$, we conclude that $Q_1 > 0$. Furthermore, we get $Q_2 = 0$ and let $Q_3 > 0$, then we obtain $Q > 0$, which is a solution of generalized Lyapunov equation (6). Obviously, the matrix $[E^T A^T]$ is of full row rank.
Suppose the matrix $A$ is nonsingular. There exists a state feedback $\xi A_{\lambda A_{\lambda A}}$. Then
\begin{equation}
\xi \text{ have corresponding to the eigenvalue, i.e. }
\end{equation}
According to Lemma 1, we have
\begin{equation}
\text{Lyapunov equation (6), we can obtain }
\end{equation}
Descriptor systems (2) are regular. Substituting matrices (18) into generalized Lyapunov equation (6) has a positive definite solution $Q$. Then state feedback law as
\begin{equation}
u(t) = -Fx(t) = -\gamma (R + B^TQB)^{-1}B^TQAx(t),
\end{equation}
where $0 < \gamma < 1$ and
\begin{equation}
\sigma(B(R + B^TQB)^{-1}B^TQ) < \frac{1}{\gamma},
\end{equation}
and $R = R^T \in R^{n \times n}$ is a positive definition matrix, where $\sigma(\bullet)$ denotes the eigenvalues of “$\bullet$”.

**Proof:** Sufficiency. We choose state feedback law (14), and assume that
\begin{equation}
\lambda = [I - \gamma B(R + B^TQB)^{-1}B^TQ]A.
\end{equation}
According to (15),
\begin{equation}
I - \gamma B(R + B^TQB)^{-1}B^TQ,
\end{equation}
is nonsingular. Then
\begin{equation}
A = [I - \gamma B(R + B^TQB)^{-1}B^TQ]^{-1}\overline{A}.
\end{equation}
When $0 = w_1 \in R$,
\begin{equation}
det(w_1E - \overline{A}) = \det(-A) \det(I - \gamma B(R + B^TQB)^{-1}B^TQ) \neq 0.
\end{equation}
Descriptor systems (2) are regular. Substituting matrices (18) into generalized Lyapunov equation (6), we can obtain
\begin{equation}
E^TQ[I - \gamma B(R + B^TQB)^{-1}B^TQ]^{-1}\overline{A}
\end{equation}
\begin{equation}
\quad + \overline{A}^T[I - \gamma B(R + B^TQB)^{-1}B^TQ]^{-1}QE = -E^TPE.
\end{equation}

Assume that $\lambda$ is an eigenvalue of matrices pair $(E, \overline{A})$, $\xi$ is an eigenvector corresponding to the eigenvalue, i.e. $\lambda E\xi = \overline{A}\xi$.

In (19) , premultiplication by $\xi^*$ and postmultiplication by $\xi$, respectively, we have
\begin{equation}
\xi^*E^TQ[I - \gamma B(R + B^TQB)^{-1}B^TQ]^{-1}\overline{A}\xi + \xi^*\overline{A}^T[I - \gamma B(R + B^TQB)^{-1}B^TQ]^{-1}QE\xi = -\xi^*E^TPE\xi.
\end{equation}
According to Lemma 1, we have
\begin{equation}
[I - \gamma B(R + B^TQB)^{-1}B^TQ]^{-1} = I + \frac{\gamma}{1 \gamma} B \left( \frac{1}{1 \gamma} R + B^TQB \right)^{-1}B^TQ,
\end{equation}
\begin{equation}
\lambda^*E^TQ[I + \frac{\gamma}{1 \gamma} B \left( \frac{1}{1 \gamma} R + B^TQB \right)^{-1}B^TQ]E\xi + \lambda^*E^T[I
\end{equation}
\begin{equation}
\quad + \frac{\gamma}{1 \gamma} QB \left( \frac{1}{1 \gamma} R + B^TQB \right)^{-1}B^TQ]E\xi = -\xi^*E^TPE\xi,
\end{equation}
\begin{equation}
2\text{Re}(\lambda)\xi^*E^T[Q + \frac{\gamma}{1 \gamma} QB \left( \frac{1}{1 \gamma} R + B^TQB \right)^{-1}B^TQ]E\xi = -\xi^*E^TPE\xi,
\end{equation}
where $\lambda^* = \overline{\lambda}$, $\xi^* = \overline{\xi}$. Since descriptor systems (2) are regular, we have $E\xi \neq 0$. Then
\begin{equation}
\xi^*E^TPE\xi > 0,
\end{equation}
\begin{equation}
\xi^*E^T[Q + \frac{\gamma}{1 \gamma} QB \left( \frac{1}{1 \gamma} R + B^TQB \right)^{-1}B^TQ]E\xi > 0.
\end{equation}
Therefore, \( \text{Re}(\lambda) < 0 \) holds. The closed-loop descriptor systems (2) are stable. Let 
\[
\overrightarrow{Q} = Q[1 - \gamma B(R + B^T Q B)^{-1} B^T Q].
\]
According to (15), we get \( \overrightarrow{Q} > 0 \). Similarly to theorem 1, we can prove descriptor 
systems (2) is impulse-free. So when state feedback law is (14), closed-loop descriptor systems (2) are admissible.

**Necessity.** Its proof is similar to theorem 1.

**Theorem 3.** Suppose matrix \( A \) is nonsingular. Then state feedback law (22) that satisfies (21) makes descriptor systems (23) are admissible if and only if for a given positive definite symmetric matrix \( P \), generalized Lyapunov equation (6) has a positive definite solution \( Q \),

\[
\sigma(BB^T Q) < \frac{1}{\gamma},
\]
the feedback law is

\[
u = -\gamma B^T Q Ax(t) \quad (\gamma > 0),
\]
and the closed-loop descriptor systems are

\[
E \frac{dx(t)}{dt} = (A - \gamma BB^T QA)x(t).
\]

**Proof:** The proof is similar to that of theorem 2 and the details are omitted.

Using matrix \( L \) in (3) to denote the actuator failures, we have the following result about the integrity for descriptor systems.

**Theorem 4.** Assume matrix \( A \) is nonsingular. Consider descriptor systems (1) with feedback law

\[
u = -\gamma B^T Q Ax(t),
\]
where matrix \( Q \) is the positive definite solution of generalized Lyapunov equation (6). When some actuator failures of descriptor systems (1) happen, the descriptor systems (4) are still admissible if

\[
\sigma(BLB^T Q) < \frac{1}{\gamma},
\]
holds, i.e. descriptor systems (1) have integrity.

**Proof:** Since

\[
E \frac{dx(t)}{dt} = (A - \gamma BB^T QA)x(t) = \tilde{A}x(t),
\]
where

\[
\tilde{A} = (I - \gamma BB^T Q)A,
\]
from (6) and (24), we can get

\[
\tilde{A}^T(I - \gamma BB^T Q)^{-T} Q E + E^T Q (I - \gamma BB^T Q)^{-1} \tilde{A} = -E^T P E.
\]
Assume that \( \tau \) is an arbitrary eigenvalue of matrices pair \((E, \tilde{A})\) and \( \eta \) is the eigenvector corresponding to \( \tau \). Premultiplication by \( \eta^* \) and postmultiplication by \( \eta \) respectively in (25), we have

\[
\tau^* \eta^* E^T (I - \gamma BB^T Q)^{-T} Q E \eta + \tau \eta^* E^T Q (I - \gamma BB^T Q)^{-1} E \eta = -\eta^* E^T P E \eta.
\]
Since

\[
(I - \gamma BB^T Q)^{-1} = I + \gamma BB^T (I - \gamma QBB^T)^{-1} Q,
\]
we have
\[
\tau \eta^* E^T (Q + \gamma Q B L B^T (I - \gamma Q B L B^T)^{-1} Q) E \eta \\
+ \tau^* \eta^* E^T (Q + \gamma Q (I - \gamma B L B^T Q)^{-1} B L B^T Q) E \eta = - \eta^* E^T P E \eta,
\]
\[
2 \Re(\tau) \eta^* E^T (Q + \gamma Q B L B^T (I - \gamma Q B L B^T)^{-1} Q) E \eta = - \eta^* E^T P E \eta.
\]
Note that
\[
\eta^* E^T P E \eta > 0,
\]
and
\[
\eta^* E^T (Q + \gamma Q B L B^T (I - \gamma Q B L B^T)^{-1} Q) E \eta > 0,
\]
we have \( \Re(\tau) < 0 \). Descriptor systems (4) are stable. Similarly, we can prove descriptor systems (4) to be regular and impulse-free. Namely, descriptor systems (1) have integrity.

**Remark 2.** When actuator failures happen in a single input and single output systems, their inputs are zero, then systems are open-loop, and its property about regularity, stability and impulse-free is determined by the system itself. In a multi-input and multi-output system, every actuator only take on part control of task. If some actuators go wrong, systems can still finish some functions by making use of the rest of the normal actuators.

Lastly, we give another description about integrity using invariant eigenvalues under certain conditions.

**Lemma 2.** For an arbitrary matrix \( P \geq 0 \), Riccati equation
\[
E^T QA + A^T QE - E^T Q B (B^T Q B + R)^{-1} B^T Q E + E^T P E = 0,
\]
has a positive semidefinite solution \( Q \), and assumption \( \lambda \) is a stable eigenvalue of matrices pair \((E, A)\), \( \xi \) is the eigenvector corresponding to \( \lambda \). If \( P E \xi = 0 \), then \( Q E \xi = 0 \).

**Proof:** Since
\[
\xi^* E^T QA \xi + \xi^* A^T QE \xi - \xi^* E^T Q B (B^T Q B + R)^{-1} B^T Q E \xi + \xi^* E^T P E \xi = 0,
\]
\[
2 \Re(\lambda) \xi^* E^T QE \xi - \xi^* E^T Q B (B^T Q B + R)^{-1} B^T Q E \xi + \xi^* E^T P E \xi = 0,
\]
\[
(-2 \Re(\lambda)) \xi^* E^T QE \xi + \xi^* E^T Q B (B^T Q B + R)^{-1} B^T Q E \xi = 0,
\]
\[
(-2 \Re(\lambda)) > 0,
\]
\[
\xi^* E^T QE \xi \geq 0,
\]
\[
\xi^* E^T Q B (B^T Q B + R)^{-1} B^T Q E \xi \geq 0.
\]
We have \( Q E \xi = 0 \). The result is true.

**Theorem 5.** If \( \lambda_1, \lambda_2, \ldots, \lambda_k \) are eigenvalues of matrices pair \((E, A)\), \( \xi_1, \xi_2, \ldots, \xi_k \) are the corresponding eigenvectors, \( \Re(\lambda_i) < 0 \) \((i = 1, 2, \ldots, k)\), \( P E \xi_i = 0 \). Then there are the same eigenvalues between \((E, A)\) and
\[
\{E, [A - \gamma B L (B^T Q B + R)^{-1} B^T Q A)]\},
\]
i.e.
\[
\lambda(E, A) = \lambda(E, A - \gamma B L (B^T Q B + R)^{-1} B^T Q A),
\]
where \(\lambda(E, A)\) denotes the eigenvalues of matrix pair \((E, A)\).

**Proof:** From Lemma 2, we have

\[
\lambda_i E \xi_i = A \xi_i,
\]

and

\[
[A - \gamma BL(B^TQB + R)^{-1}B^TQA] \xi_i = A \xi_i - \gamma \lambda_i BL(B^TQB + R)^{-1}B^TQA \xi_i = A \xi_i - \gamma \lambda_i BLB \xi_i = A \xi_i - \gamma \lambda_i B \xi_i = \lambda_i E \xi_i.
\]

4. Illustrative examples

Consider descriptor systems (1) with matrices

\[
E = \begin{bmatrix}
-1.75 & 7.75 & -13.75 \\
-5.5 & 16.5 & -27.5 \\
-2.75 & 7.75 & -12.75
\end{bmatrix},
A = \begin{bmatrix}
5.3895 & -4.9227 & 8.8910 \\
6.6306 & -6.6368 & 14.0339 \\
-0.3308 & -0.0974 & 1.7473
\end{bmatrix},
B = \begin{bmatrix}
-12.643 & -8.7481 \\
-21.4219 & -15.1159 \\
-4.4699 & -3.7774
\end{bmatrix},
P = \begin{bmatrix}
9.1650 & -6.75 & 5.5875 \\
-6.75 & 6.3325 & -8.4375 \\
5.5875 & -8.4375 & 17.25
\end{bmatrix}.
\]

Matrix A is nonsingular. Solving generalized Lyapunov equation (6), we obtain

\[
Q = \begin{bmatrix}
36.4775 & -22.9 & 17.0875 \\
-22.9 & 14.8825 & -12.2125 \\
17.0875 & -12.2125 & 12.9375
\end{bmatrix}.
\]

Let \(\gamma = 0.001\), then matrices \((I - \gamma BB^TQ)\) and \((I - \gamma BLB^TQ)\) are nonsingular. When there are no fault for descriptor systems (1), the closed-loop systems are regular and impulse-free, and the eigenvalue set is

\[
\lambda(E, (I - \gamma BB^TQ)A) = \{-1.0622, -0.4128\} \subset \sigma^-,
\]

where \(\sigma^-\) denotes the open left half plane of the complex plane. So the closed-loop descriptor systems are admissible. When some actuator failures happen, we assume that the failure matrix is \(L = \text{diag}[\beta_1, \beta_2]\) (where \(\beta_i \in [0, 1], i = 1, 2\)). The closed-loop the closed-loop systems are also regular and impulse-free. According to

\[
\det(sE - (I - \gamma BLB^TQ)A) = 0,
\]

we have

\[
(100s + 100 - 4.8\beta_1 - \beta_2)(100s + 50 + 1.05\beta_1 + 2.25\beta_2) + (-250 + 3.15\beta_1 + 2.25\beta_2)(1.6\beta_1 + \beta_2) = 0.
\]

If \(s = bi\) \((i^2 = -1, b \in R)\) is the solution of equation (28), we have

\[
\Delta + 100b(150 - 3.75\beta_1 + 1.15\beta_2)i = 0,
\]

where

\[
\Delta = (100 - 4.8\beta_1 - \beta_2)(50 + 1.05\beta_1 + 2.25\beta_2) - 10^4b^2 + (-250 + 3.15\beta_1 + 2.25\beta_2)(1.6\beta_1 + \beta_2).
\]

From (29), we get

\[
100b(150 - 3.75\beta_1 + 1.15\beta_2) = 0,
\]

and

\[
\Delta = 0.
\]
From (31), we obtain \( b = 0((150 - 3.75/\beta_1 + 1.15/\beta_2) > 145 > 0) \). From (30), we have
\[
\Delta > 94 \times 50 - 250 \times 3 - 10^4 b^2 > 10^3 - 10^4 b^2 = 10^3 > 0.
\]
It is in contradiction with (32). So there are not any roots on the imaginary axis of equation (28) for arbitrary continuous \( \beta_i \in [0, 1], (i = 1, 2) \). According to (27), it is impossible that the roots of equation (28) go through open right half complex plane or reach imaginary axis for continuous \( \beta_i \). Therefore we ascertain that the descriptor systems (1) possess integrity.

5. Conclusion

In this short paper, we proposed an approach to design integrity controllers for descriptor systems. A positive definite solution of generalized Lyapunov equation is utilized for determining the state feedback control to make descriptor systems (1) integrable.

References


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