UNITARY CYCLIC DOA ALGORITHMS FOR COHERENT CYCLOSTATIONARY SIGNALS

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Abstract. Two signal selective Unitary Cyclic DOA algorithms are presented by introducing a new forward-backward smoothed covariance matrix. Compared with conventional Cyclic DOA algorithms, the suggested approaches have better performance in the presence of multipath propagation. In addition, these approaches not only allow to select desired signals and to ignore interferences by exploiting the cyclostationarity property of signals of interest (SOIs), but also reduce the computational complexity by real-valued eigendecomposition. Simulation results that illustrate the performance of these approaches in conjunction with conventional Cyclic DOA algorithms are described.

Key Words. Array signal processing, cyclostationarity, DOA, MUSIC, ESPRIT.

1. Introduction

Array signal processing techniques basically rely on the spatial properties of the signals impinging on the array of sensors. In applications such as radar, sonar, signal intelligence, or communications, more specifically, almost all man-made communication signals exhibit the cyclostationarity property which can be exploited in signal processing to signify desired signals and to discriminate against interferences and noise [1]-[6].

The cyclostationarity has been first introduced into array signal processing by Gardner [4]. Compared with conventional MUSIC [7] and ESPRIT [8] algorithms, Cyclic MUSIC [2]-[3] and Cyclic ESPRIT [4] have three advantages: firstly the total number of signals impinging on the array, including both signals of interest (SOIs) and interference, is larger than the number of sensors and although the characteristics of interfering signals are unknown, their effects can be subtracted; secondly it is possible to resolve two signals spaced more closely than the resolution threshold of the array when only one signal is a SOI; thirdly the noise characteristics of the sensors and the environment may be unknown or they may not be accurately modelled as independent and identically distributed Gaussian random processes.

However when coherent or highly correlated signals are present, these cyclic algorithms perform poorly. Thus two signal selective Unitary Cyclic DOA algorithms are proposed in this paper to circumvent the drawback by incorporating forward-backward averaging, which have better performance in the presence of multipath propagation. In the presence of additive noise, the computation of a signal subspace estimate requires a singular value decomposition (SVD) or an eigenvalue decomposition (EVD), which is computationally expensive, since $o(M^3)$ operations are necessary to update the SVD or EVD if a new sample vector of dimension $M$ arrives. Unitary Cyclic DOA algorithms achieve a substantial reduction of the computational complexity by mapping centro-Hermitian matrices to real matrices [9]-[11]. Thus, these algorithms not only
reduce the computational complexity by real-valued eigendecomposition, but also allow to select desired signals and to ignore interferences by exploiting the cyclostationarity property of the SOIs.

This paper is organized as follows. Section 2 briefly describes the narrow band data model of the cyclostationary signals. And conventional Cyclic DOA algorithms are reviewed in Section 3. By reconstructing a new forward-backward smoothed covariance matrix, two Unitary Cyclic DOA algorithms are given in Section 4. Finally, simulation results are provided in Section 5, followed by a conclusion in Section 6.

2. Signal model

Consider a Uniform Linear Array (ULA) composed of \( m \) omnidirectional sensors. Suppose \( d \) narrow band far-field sources with center frequency \( \omega_0 \) impinging from the directions \( \theta_1, \ldots, \theta_d \). Assume that there are \( N \) snapshots \( x(1), x(2), \ldots, x(N) \) available, the observation vector can be modelled as

\[
x(k) = A s(k) + i(k) + n(k),
\]

where

\[
A = [a(\theta_1), \ldots, a(\theta_d)]
\]
is a \( m \times d \) matrix of the signal direction vectors, and \( a(\theta) \) is a \( m \times 1 \) steering vector. \( s(k) = [s_1(k), \ldots, s_d(k)]^T \) is a vector of cyclostationary signals with cycle frequency \( \alpha \). \( i(k) \) is a vector of interfering sources with cyclostationary property, and \( n(k) \) is a vector of sensor noise. Hence for some cyclic frequency \( \alpha \) and some lag parameter \( \tau \), the cyclic autocorrelation matrix of the observation vector is defined by

\[
R_{\alpha}(\tau) = \langle x(k) x^H(k + \tau) e^{-j2\alpha \tau k} \rangle = AR_{\alpha}(\tau)A^H
\]

where

\[
R_{\alpha}(\tau) = \langle s(k) s^H(k + \tau) e^{-j2\alpha \tau k} \rangle
\]
is a \( d \times d \) cyclic autocorrelation matrix of the cyclostationary signal vector. \( \langle \cdot \rangle \) denotes the finite time average operator, and the superscript \( H \) denotes complex conjugate transpose of a vector or matrix.

3. Cyclic DOA algorithms

The cyclic autocorrelation matrix at cycle frequency \( \alpha \) for some lag parameter \( \tau \) is nonzero and can be estimated by

\[
R_{\alpha}(\tau) = \frac{1}{N} \sum_{k=1}^{N} x(k) x^H(k + \tau) e^{-j2\alpha \tau k}
\]

where depending on the type of modulation used, the cycle frequency \( \alpha \) is usually equal to twice the carrier frequency, multiple of the baud rate, spreading codes repetition rate, chip rate or combinations of these. Compared with the covariance matrix exploited by the MUSIC algorithm, the cyclic autocorrelation matrix exploited by the Cyclic MUSIC method is generally not Hermitian. Then, instead of using the eigenvalue decomposition (EVD), Cyclic MUSIC algorithm uses the singular value
where the subscripts $s$ and $n$ stand for signal- and null-subspaces, respectively. Thus the Cyclic MUSIC spatial spectrum can be written as

$$P_{CM}(\theta) = \frac{\|a(\theta)\|^2}{\|U_n^s a(\theta)\|^2}$$

**Algorithm 1.** Cyclic MUSIC Algorithm

*Step 1:* Choose $\alpha$ to be a cycle frequency of the desired signals and the delay parameter $\tau$.

*Step 2:* Find the null space $U_n$ of $R^\alpha(\tau)$ by singular value decomposition and its rank $Z$.

*Step 3:* Determine the number of SOIs, $d = m - Z$.

*Step 4:* Search over $\theta$ for the $d$ highest peaks in $P_{CM}(\theta)$.

From the structure of steering vector matrix $A$ [12], it can be decomposed into $A_1, A_2 \in \mathbb{C}^{(n-1) \times d}$ so that two matrices are related by the unique nonsingular transformation diagonal matrix $\Phi$, that is

$$A_2 = A_1 \Phi$$

where $\Phi = \text{diag}\{e^{-j\phi_1}, e^{-j\phi_2}, \ldots, e^{-j\phi_d}\}$ is a rotation operator. Similarly, two matrices $U_1$ and $U_2$ are obtained from the signal subspace matrix $U_s$ and related by a unique nonsingular transformation matrix $\Psi$. When the cyclic autocorrelation matrix, $R^\alpha(\tau)$, of the cyclostationary signal vector is nonsingular, $A$ and $U_s$ share a common column space and are related by another unique nonsingular transformation matrix $T$. Thus

$$U_1 = A_1 T$$
$$U_2 = A_2 T$$

Substituting for $U_1$ and $U_2$ and use the fact that $A$ is of full rank, one obtains

$$T \Psi T^{-1} = \Phi$$

which states that the eigenvalues of $\Psi$ are equal to the diagonal elements of $\Phi$ and that the columns of $T$ are eigenvectors of $\Psi$. An eigendecomposition of $\Psi$ gives its eigenvalues and by equating them to $\Phi$ leads to the DOA estimates.

For finite number of time samples, the algorithm can be summarized as follows

**Algorithm 2.** Cyclic ESPRIT Algorithm

*Step 1:* Choose the cycle frequency of desired signals $\alpha$ and the lag parameter $\tau$.

*Step 2:* Estimate the cyclic autocorrelation matrix $R^\alpha(\tau)$ from the received data of ULA.

*Step 3:* Find the signal subspace $U_s$ of the cyclic autocorrelation matrix $R^\alpha(\tau)$, and detect the number $d$ of SOIs based on MDL principle.

*Step 4:* Based on equations (8) and (9), form $U_1$ and $U_2$.

*Step 5:* Calculate the eigenvalues $\phi_k(k = 1, \ldots, d)$ of matrix pencil $\{U_1, U_2\}$.

*Step 6:* Estimate DOAs of SOIs by these eigenvalues.

4. Unitary Cyclic DOA algorithms

Let a ULA with $L$ identical sensors $\{1, \ldots, L\}$ divided into overlapping subarrays of
size $m$, with sensors $\{1, \ldots, m\}$ forming the first subarray, sensors $\{2, \ldots, m+1\}$ forming the second subarray, etc. Thus the vector of received signals at the $p$th subarray can be written as

$$x_p(k) = A\Phi^{(p-1)}s(k) + i_p(k) + n_p(k)$$

**Lemma 1.** A complex matrix $\Gamma \in \mathbb{C}^{m \times m}$ is called Hermitian if $\Gamma = \Gamma^H G$ where $G$ denotes a complex matrix of size $m \times m$.

From the Lemma 1 and equation (3), assume that $G = R^a_{\alpha}(\tau)$, and we obtain the following Hermitian matrix

$$\Gamma = R^a_{\alpha}(\tau)[R^a_{\alpha}(\tau)]^H = A\Gamma^a_{\alpha}(\tau)A^H$$

where the rank of $\Gamma^a_{\alpha}(\tau)$ is $d$. Using the eigenvalue decomposition, the matrix $\Gamma$ is decomposed to be

$$\Gamma = [\hat{U}_s \hat{U}_n] \left[ \begin{array}{cc} \Gamma_s & 0 \\ 0 & \Gamma_n \end{array} \right] [\hat{U}_s \hat{U}_n]^H$$

**Theorem 1.** Assume that $U_s$ denotes the left signal space matrix of the cyclic autocorrelation matrix $R^a_{\alpha}(\tau)$ and $\hat{U}_s$ the signal space matrix of the Hermitian matrix $\Gamma$, then two matrices share a common column space.

**Proof.** Assume that $\hat{U}_n$ is the null space matrix of the Hermitian matrix $\Gamma$, then

$$A\Gamma^a_{\alpha}(\tau)A^H \hat{U}_n = 0$$

$$\hat{U}_s^H \hat{U}_n = 0$$

Obviously, $\hat{U}_s$ and the steering matrix $A$ share the same column space, that is

$$R(\hat{U}_s) = R(A)$$

Similarly, for the cyclic autocorrelation matrix $R^a_{\alpha}(\tau)$, the same important equation is obtained as

$$R(\hat{U}_s) = R(A)$$

Finally, $U_s$ and $\hat{U}_s$ share the same signal space.

Thus, the forward covariance matrix of the $p$th subarray is defined by

$$R^a_p(\tau) = A\Phi^{(p-1)}S^a_p(\tau)[\Phi^{(p-1)}]^H A^H$$

where $S^a_p(\tau)$ is given by

$$S^a_p(\tau) = R^a_{\alpha}(\tau)[\Phi^{(p-1)}]^H A^H A\Phi^{(p-1)}[R^a_{\alpha}(\tau)]^H$$

and the backward covariance matrix of the $p$th subarray is defined as follows

$$\tilde{R}^a_p(\tau) = JA^*[\Phi^{(p-1)}][S^a_p(\tau)][\Phi^{(p-1)}]^H A^*J$$

$$= A\Phi^{(2-m-p)}[S^a_p(\tau)][\Phi^{(2-m-p)}]^H A^H$$

where $J \in \mathbb{C}^{m \times m}$ is the exchange matrix with ones on its antidiagonal and zeros elsewhere and the superscript $*$ denotes complex conjugation. So the forward-backward smoothed covariance matrix is introduced by
or more compactly as

\[
\tilde{R}_{FB}^\alpha(\tau) = AS_F^\alpha(\tau)A^H
\]

where \( S_F^\alpha(\tau) \), the modified cyclic autocorrelation matrix \([13][14]\) of the cyclostationary signals, is given by

\[
S_F^\alpha(\tau) = \frac{1}{2P} \sum_{p=1}^{P} (\Phi^{(p-1)}S_p^\alpha(\tau)[\Phi^{(p-1)}]^H + \Phi^{(2-n-p)}[S_p^\alpha(\tau)][\Phi^{(2-n-p)}]^H)
\]

**Theorem 2.** The modified cyclic autocorrelation matrix of the array \( \tilde{R}_{FB}^\alpha(\tau) \) is centro-Hermitian.

*Proof. First, by multiplying the complex conjugation form of the modified cyclic autocorrelation matrix \( \tilde{R}_{FB}^\alpha(\tau) \) on both sides with the exchange matrix \( J \), we obtain a matrix in the following form:

\[
J[\tilde{R}_{FB}^\alpha(\tau)]^J = JA^*[S_F^\alpha(\tau)][JA^*]^H
\]

where the complex conjugation form of \( S_F^\alpha(\tau) \) is rewritten as

\[
[S_F^\alpha(\tau)']^J = \frac{1}{2P} \sum_{p=1}^{P} (\Phi^{(p-1)}S_p^\alpha(\tau)[\Phi^{(p-1)}]^H + \Phi^{(2-n-p)}[S_p^\alpha(\tau)][\Phi^{(2-n-p)}]^H)^*
\]

Obviously, using (23) and (24) the important equation can be drawn as

\[
J[\tilde{R}_{FB}^\alpha(\tau)]^J = \tilde{R}_{FB}^\alpha(\tau)
\]

Secondly, a complex matrix \( G \) is called centro-Hermitian\([9]-[11]\) if \( G = JG^*J \). Thus, the modified cyclic autocorrelation matrix \( \tilde{R}_{FB}^\alpha(\tau) \) is centro-Hermitian.

4.1 Real-valued realization

By exploiting the centro-Hermitian property of \( \tilde{R}_{FB}^\alpha(\tau) \), we introduce the real-valued covariance matrix as

\[
C = Q^H \tilde{R}_{FB}^\alpha(\tau)Q
\]

where \( Q \) is any unitary column conjugate symmetric matrix, for example

\[
Q = \frac{1}{\sqrt{2}} \begin{bmatrix} I & jI \\ J & -jJ \end{bmatrix}
\]

and

\[
Q = \frac{1}{\sqrt{2}} \begin{bmatrix} I & 0 & jI \\ 0^T & \sqrt{2} & 0^T \\ J & 0 & -jJ \end{bmatrix}
\]

can be chosen for arrays with an even and odd number of sensors, respectively, where
\( \mathbf{I} \) is an identity matrix and \( \mathbf{0} \) is a vector \([0,0,\ldots,0]^T\). Compared with the modified cyclic autocorrelation matrix (21), the real-valued covariance matrix is given by

\[ \mathbf{C} = \tilde{\mathbf{A}} S_{FB}^\alpha (\tau) \tilde{\mathbf{A}}^H \]

where

\[ \tilde{\mathbf{A}} = \mathbf{Q}^H \mathbf{A} \]

denotes the relationship between the former and new manifolds.

Contrary to the cyclic covariance matrix \( \mathbf{R}^\alpha_\eta(\tau) \) exploited by Cyclic MUSIC algorithm, the forward-backward smoothed covariance matrix \( \tilde{\mathbf{R}}_{FB}^\alpha(\tau) \) presented here is Hermitian. Let the eigendecompositions of the matrices (20) and (26) be defined as

\[ \tilde{\mathbf{R}}_{FB}^\alpha(\tau) = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H = \mathbf{U}_s \mathbf{A}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{A}_n \mathbf{U}_n^H \]

\[ \mathbf{C} = \mathbf{E} \mathbf{\Gamma} \mathbf{E}^H = \mathbf{E}_s \mathbf{\Gamma}_s \mathbf{E}_s^H + \mathbf{E}_n \mathbf{\Gamma}_n \mathbf{E}_n^H \]

where

\[ \mathbf{U}_s = [u_1, \ldots, u_d], \mathbf{U}_n = [u_{d+1}, \ldots, u_m] \]

\[ \mathbf{A}_s = \text{diag}\{\lambda_1, \ldots, \lambda_d\}, \mathbf{A}_n = \text{diag}\{\lambda_{d+1}, \ldots, \lambda_m\} \]

\[ \mathbf{E}_s = [\varepsilon_1, \ldots, \varepsilon_d], \mathbf{E}_n = [\varepsilon_{d+1}, \ldots, \varepsilon_m] \]

\[ \mathbf{\Gamma}_s = \text{diag}\{\gamma_1, \ldots, \gamma_d\}, \mathbf{\Gamma}_n = \text{diag}\{\gamma_{d+1}, \ldots, \gamma_m\} \]

and the subscripts \( s \) and \( n \) stand for signal- and null-spaces, respectively. The characteristic equation for the matrix (20) is

\[ \tilde{\mathbf{R}}_{FB}^\alpha(\tau) \mathbf{u} = \lambda \mathbf{u} \]

which can be further rewritten as

\[ \mathbf{Q}^H \tilde{\mathbf{R}}_{FB}^\alpha(\tau) \mathbf{u} = \mathbf{Q}^H \tilde{\mathbf{R}}_{FB}^\alpha(\tau) \mathbf{Q}^H \mathbf{u} \]

\[ = \mathbf{Q}^H \mathbf{u} \]

\[ = \mathbf{Q}^H \mathbf{u} \]

Equation (32) can be identified as the characteristic equation for the real-valued covariance matrix (26). Hence, the eigenvectors and eigenvalues of the matrices (20) and (26) are related as

\[ \mathbf{E} = \mathbf{Q}^H \mathbf{U} \]

\[ \mathbf{\Gamma} = \mathbf{\Lambda} \]

By simple manipulation with the conventional MUSIC algorithm, the spatial spectrum of Unitary Cyclic MUSIC algorithm is obtained as

\[ P_{\text{ECM}}(\theta) = \frac{\mathbf{\hat{a}}^H(\theta) \mathbf{Q}^H \mathbf{a}(\theta)}{\mathbf{\hat{a}}^H(\theta) \mathbf{Q}^H \mathbf{U}_s \mathbf{U}_s^H \mathbf{Q}^H \mathbf{a}(\theta)} \]

\[ = \frac{\mathbf{\hat{a}}^H(\theta) \mathbf{\bar{a}}(\theta)}{\mathbf{\hat{a}}^H(\theta) \mathbf{Q}^H \mathbf{U}_s \mathbf{U}_s^H \mathbf{Q} \mathbf{\bar{a}}(\theta)} \]

\[ = \frac{\mathbf{\hat{a}}^H(\theta) \mathbf{\bar{a}}(\theta)}{\mathbf{\hat{a}}^H(\theta) \mathbf{E}_s \mathbf{E}_s^H \mathbf{\bar{a}}(\theta)} \]

which can be simplified to
Algorithm 3. Unitary Cyclic MUSIC Algorithm

Step 1: Choose the cycle frequency $\alpha$ of desired signals and the lag parameter $\tau$;
Step 2: Estimate the forward backward smoothed covariance matrix $\hat{R}_{\alpha\tau}^\alpha$ from the received data of ULA;
Step 3: Form the real-valued matrix $C$ using (26);
Step 4: Find the null subspace $E_s$ of the real-valued matrix $C$, and detect the number $d$ of SOIs based on MDL principle;
Step 5: Search over $\theta$ for the $d$ highest peaks in $P_{\text{UCM}}(\theta)$.

Using equations (8)-(10), the important relationship between $A$ and $E_s$ is obtained as

$$R(A) = R(\tilde{E}_s)$$

where

$$\tilde{E}_s = QE_s$$

In addition, in conjunction with $U_1$ and $U_2$, $\tilde{E}_1$ and $\tilde{E}_2$ are extracted from the matrix $\tilde{E}_s$.

Algorithm 4. Unitary Cyclic ESPRIT Algorithm

Step 1: Choose the cycle frequency $\alpha$ and the lag parameter $\tau$;
Step 2: Estimate the forward backward smoothed co-variance matrix $\hat{R}_{\alpha\tau}^\alpha$ from the received data of ULA;
Step 3: Form the real-valued matrix $C$ with the use of (25);
Step 4: Find the signal subspace $E_s$ of the real-valued matrix $C$, and detect the number $d$ of SOIs based on MDL principle;
Step 5: Based on equations (31) and (33), form $\tilde{E}_1$ and $\tilde{E}_2$;
Step 6: Calculate these eigenvalues $\phi_k(k = 1, \ldots, d)$ of matrix pencil $\{\tilde{E}_1, \tilde{E}_2\}$;
Step 7: Estimate DOAs of SOIs by these eigenvalues.

4.2 Estimating the number of sources

Many of the Cyclic direction-finding methods require the number of SOIs, and their performance is dependent on the perfect knowledge of these numbers. But correlation of SOIs may exist due to multipath propagation, and it tends to reduce the rank of the cyclic autocorrelation matrix. Using the spatial smoothing method, the modified cyclic autocorrelation matrix $\hat{R}_{\alpha\tau}^\alpha$ is given for resoving this rank deficiency. Then the number of cyclostationary signals is determined by these algorithms such as VCBM, MDL[1], etc.

5. Simulation results

In this section, we present some simulation results to show the behavior of Unitary Cyclic DOA algorithms and to compare them with the conventional Cyclic DOA
algorithm. Assume a ULA with eight omnidirectional sensors spaced by a half wavelength of the coming signals. Incoming cyclostationary signals with central frequency 0.1 are generated with noise at cycle frequency 0.2 and the signal-to-noise ratio (SNR) is 10 dB for each signal.

5.1 Unitary Cyclic MUSIC algorithm

In the first simulation, the signal selectivity and accuracy of Unitary Cyclic MUSIC algorithm are tested. Three uncorrelated SOIs arrive from $-30^\circ$, $-10^\circ$ and $5^\circ$. The resulting spatial spectra are shown in Fig.1.

In the second simulation, the signal selectivity and accuracy of Unitary Cyclic MUSIC algorithm are tested. Two uncorrelated SOIs arrive from $-15^\circ$ and $-25^\circ$, and one interferer arrives from $15^\circ$. The resulting spatial spectra are shown in Fig.2.

In the third simulation, Unitary Cyclic MUSIC algorithm accurately estimates three SOI DOAs in multipath propagation. Three coherent SOIs arrive from $-30^\circ$, $-10^\circ$ and $5^\circ$. The resulting spatial spectra are shown in Fig.3.

In the fourth simulation, Unitary Cyclic MUSIC algorithm accurately estimates two SOI DOAs in multipath propagation. Two coherent SOIs arrive from $-15^\circ$ and $-25^\circ$, one interferer arrives from $15^\circ$. The resulting spatial spectra are shown in Fig.4.

In the fifth simulation, the performance of Unitary Cyclic MUSIC algorithm is affected by SNR. Two uncorrelated signals arrive from $-15^\circ$ and $-25^\circ$. The results for Cyclic MUSIC algorithm and Unitary Cyclic MUSIC algorithm versus the SNR are plotted in Fig.5.

In the sixth simulation, the performance of Unitary Cyclic MUSIC algorithm is affected by SNR in multipath propagation. Two coherent signals arrive from $-15^\circ$ and $-25^\circ$. The results for Cyclic MUSIC algorithm and Unitary Cyclic MUSIC algorithm versus the SNR are plotted in Fig.6.

From Figs.1-4, both Unitary Cyclic MUSIC algorithm and Cyclic MUSIC algorithm can separate uncorrelated signals, but only Unitary Cyclic MUSIC algorithm can separate coherent signals. As expected, the presence of the interferer has little effect on the SOIs. Figs.5-6 show how the SNR affects the DOA estimation. The performance of the method is quantified by the root-mean-square-error (RMSE) of 100 independent DOA estimates. Both for correlated and uncorrelated source scenarios, Unitary Cyclic MUSIC algorithm performs better than Cyclic MUSIC algorithm.

5.2 Unitary Cyclic ESPRIT algorithm

In the seventh simulation, the DOA estimate distribution of Cyclic ESPRIT algorithm is given for uncorrelated signals. Three uncorrelated SOIs arrive from $-20^\circ$, $10^\circ$ and $50^\circ$ with an SNR of 0dB. The results of 20 trial runs with $N=1000$ snapshots are plotted in Fig.7. Note that the circle ($\circ$) denotes the position of the estimated value of Cyclic ESPRIT algorithm.

In the eighth simulation, the DOA estimate distribution of Unitary Cyclic ESPRIT algorithm is given for uncorrelated signals. Three uncorrelated SOIs arrive from $-20^\circ$, $10^\circ$ and $50^\circ$ with an SNR of 0dB. The results of 20 trial runs with $N=1000$ snapshots are plotted in Fig.8. Note that the star ($\ast$) denotes the position of the estimated value of
Unitary Cyclic ESPRIT algorithm.

In the ninth simulation, the DOA estimate distribution of Cyclic ESPRIT algorithm is given for coherent signals. Three coherent SOIs arrive from $-20^\circ$, $10^\circ$ and $50^\circ$ with an SNR of 0dB. The results of 20 trial runs with $N=1000$ snapshots are plotted in Fig.9.

In the tenth simulation, the DOA estimate distribution of Unitary Cyclic ESPRIT algorithm is given for coherent signals. Three coherent SOIs arrive from $-20^\circ$, $10^\circ$ and $50^\circ$ with an SNR of 0dB. The results of 20 trial runs with $N=1000$ snapshots are plotted in Fig.10.

In the eleventh simulation, the performance of Unitary Cyclic ESPRIT algorithm is affected by SNR. Two uncorrelated SOIs arrive from $-15^\circ$ and $-25^\circ$. The results of 200 trial runs with $N=1000$ snapshots are shown in Fig.11.

In the twelfth simulation, the performance of Unitary Cyclic ESPRIT algorithm is affected by SNR in the presence of interfering. Two uncorrelated SOIs arrive from $-15^\circ$ and $-25^\circ$, and one interferer arrives from $15^\circ$. The results of 200 trial runs with $N=1000$ snapshots are shown in Fig.12.

In the thirteenth simulation, the performance of Unitary Cyclic ESPRIT algorithm is affected by SNR for coherent signals. Two coherent signals arrive from $-15^\circ$ and $-25^\circ$. The results of 200 trial runs with $N=1000$ snapshots are shown in Fig.13.

In the fourteenth simulation, the performance of Unitary Cyclic ESPRIT algorithm is affected by SNR in the presence of interfering for coherent signals. Two coherent signals arrive from $-15^\circ$ and $-25^\circ$, and one interferer arrives from $15^\circ$. The results of 200 trial runs with $N=1000$ snapshots are shown in Fig.14.

In general, figs.7-8 show that the DOA estimate distribution of Unitary Cyclic ESPRIT algorithm has more concentration than that of Cyclic ESPRIT algorithm for uncorrelated signals with a fixed SNR. For coherent signals Figs.9-10 show that the DOA estimate distribution of Unitary Cyclic ESPRIT algorithm still has good convergence with a fixed SNR, but Cyclic ESPRIT algorithm can't separate coherent signals. Figs.11-12 show how the SNR affects the DOA estimation for uncorrelated signals. And Figs.13-14 show how the SNR affects the DOA estimation for coherent signals. The performance of the method is quantified by the root-mean-square-error (RMSE) of 200 trial runs of independent DOA estimates. Unitary Cyclic ESPRIT algorithm and Cyclic ESPRIT algorithm can separate uncorrelated signals, but only Unitary Cyclic ESPRIT algorithm can separate coherent signals. As expected, the presence of the interfering has little effect on the SOIs. Therefore Unitary Cyclic ESPRIT algorithm performs better than the Cyclic ESPRIT algorithm, both for correlated and uncorrelated source scenarios.

6. Conclusion

Unitary Cyclic DOA algorithms are proposed by constructing a new forward-backward smoothed covariance matrix in this paper. Simulation results suggest that these two proposed approaches have better signal selectivity and better resolution power than conventional Cyclic DOA algorithms, both for correlated and uncorrelated source scenarios. At the same time these approaches have low computational complexity because of real-valued computation.
Figure 1. Spatial spectra for environment containing three uncorrelated SOIs with $-30^\circ$, $-10^\circ$ and $5^\circ$ DOA.

Figure 2. Spatial spectra for environment containing two uncorrelated SOIs with $-15^\circ$ and $-25^\circ$ DOA and one interferer with $15^\circ$ DOA.
Figure 3. Spatial spectra for environment containing three coherent SOIs with -30°, -10° and 5° DOA.

Figure 4. Spatial spectra for environment containing two coherent SOIs with -15° and -25° DOA and one interferer with 15° DOA.

Figure 5. RMSE versus SNR for environment containing two uncorrelated SOIs with -15° and -25° DOA.
Figure 6. RMSE versus SNR for environment containing two coherent SOIs with $-15^\circ$ and $-25^\circ$ DOA.

Figure 7. Three uncorrelated SOIs with $-20^\circ$, $10^\circ$ and $50^\circ$ DOA.

Figure 8. Three uncorrelated SOIs with $-20^\circ$, $10^\circ$ and $50^\circ$ DOA.
Figure 9. Three coherent SOIs with $-20^\circ$, $10^\circ$ and $50^\circ$ DOA.

Figure 10. Three coherent SOIs with $-20^\circ$, $10^\circ$ and $50^\circ$ DOA.

Figure 11. Spatial spectra for environment containing two uncorrelated SOIs with $-15^\circ$ and $-25^\circ$ DOA.
Figure 12. Spatial spectra for environment containing two uncorrelated SOIs with $-15^\circ$ and $-25^\circ$ DOA and one interferer with $15^\circ$ DOA.

Figure 13. Spatial spectra for environment containing two coherent SOIs with $-15^\circ$ and $-25^\circ$ DOA.

Figure 14. Spatial spectra for environment containing two coherent SOIs with $-15^\circ$ and $-25^\circ$ DOA and one interferer with $15^\circ$ DOA.
References


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