A TWO COMPARTMENT MODEL OF A CA1 PYRAMIDAL NEURON

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ABSTRACT. We develop a two compartment, conductance based model for a pyramidal cell in the CA1 region of the hippocampus. One compartment represents the soma and proximal dendrites, while the second represents the distal dendrites. Using numerical simulations we show that our model reproduces various experimentally observed behaviours including a burst of action potentials when the distal dendrite compartment is stimulated. Finally, we study the synchronization of two such neuronal models, coupled by AMPA synapses.

1 Introduction Focal epilepsy is a neurological disorder which affects approximately 50 million people of all ages worldwide [13]. It is identified by recurrent unprovoked seizures, which are characterized by the excessive discharge (and often synchronization) of a large group of neurons in the brain. A simple, biologically relevant mathematical model would be helpful to examine the dynamics involved in these excessive discharges. Therefore we created a two-compartment mathematical model (based on a reduction of Traub et al.’s [9] 19-compartment model) of a pyramidal neuron in the CA1 region of the hippocampus, a region which is often associated with seizure activity [1, 11]. We model the communication between neurons through an AMPA synapse, which according to Traub et al. [9] is the dominant method of synchronization amongst the neurons. Using our model, we reproduce behaviour of the 19-compartment model of Traub et al. [9] and of experiments.

2 Single cell model [9] created a 19-compartment model of a CA1 pyramidal neuron, and used their model to simulate key characteristics of a CA1 neuron. However, a more simplified model may be useful to examine the effects of key parameters, to create a network model, or

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To reduce the computational demand in order to extend the model in other ways. To reduce the number of compartments in the model, we noted that the CA1 neuron exhibited trains of action potentials when a small current ($< 1 \text{nA}$) was applied to the soma, but a full calcium spike and burst followed by trains of action potentials when the same small current was applied to the distal dendrites ($0.6\lambda$ from soma) [9]. Thus we recognized the importance of distinguishing between the electrical properties in different parts of the neuron. This required the spatial separation of different channel types and the proper current flow between them [9]. We concluded our model needed at least two compartments.

To construct these compartments we observed that the ionic channels were segregated such that their conductance strength varied significantly in the distal dendrites compared to the proximal dendrites and the somatic compartment [9]. Therefore, we used the general approach of Pinsky and Rinzel [6]: we lumped Traub’s soma and proximal dendrite compartments into one soma compartment, and the distal dendrite compartments into one dendrite compartment. To determine which ion currents dominated in each compartmental region, we examined the non-unique distribution of ion channel conductance densities determined by [9]. Since we maintained the same volume/area ratio as Traub, and Traub’s compartments had a length of $0.1\lambda$, this gave our lumped compartments a length greater than $0.5\lambda$. Thus, as for Pinsky & Rinzel’s CA3 neuron model, our model should be considered as phenomenological rather than physiological.

A schematic representation of our model is given in Figure 1. The schematic shows the ionic currents present in each compartment, with the direction of current flow represented by an arrow. The applied currents ($I_D$, $I_S$) are shown, as well as the synaptic current ($I_{SYN}$). The two compartments are connected through the coupling parameters $g_C$, the strength of coupling, and $p$, the percentage of the cell model’s total area taken up by the somatic compartment. The difference in potential across the membrane is denoted by $V_S$ and $V_D$ for the somatic and dendritic compartments respectively, and represents the deviation (in mV) from the resting membrane potential of $-60 \text{ mV}$. The somatic compartment has five ionic current channels: sodium and calcium are the inward currents ($I_{Na,S}$ and $I_{Ca,S}$ respectively), and the outward currents are the delayed rectifier potassium current ($I_{K-DR,S}$), long-duration calcium-dependent AHP potassium current ($I_{K-AHP,S}$), and short-duration voltage and calcium-dependent potassium ($I_{K-C,S}$). The dendrite compartment has three ionic current channels: an inward calcium current ($I_{Ca,D}$), and the outward $I_{K-AHP,D}$ and $I_{K-C,D}$. Al-
though not shown in the schematic, there is a small leak current in both the soma and dendrite compartments. We use $C_m$ to represent the membrane capacitance in units $\mu F/cm^2$. All currents in this model have the units $\mu A/cm^2$ and all conductances have the units $mS/cm^2$.

![Schematic representation of our two-compartment CA1 neuron model.](image)

**FIGURE 1:** A schematic representation of our two-compartment CA1 neuron model.

The ionic currents are modeled as a function of their maximal conductance strengths and their corresponding Hodgkin-Huxley-like gating variables. The voltage dependent currents are given by:

\[ I_{\text{leak},j}(V_j) = g_{\text{leak},j}(V_j - V_{\text{leak}}) \]
\[ I_{\text{Na},j}(V_j, h_j) = g_{\text{Na},j} m_j^2 h_j (V_j - V_{\text{Na}}) \]
\[ I_{\text{K-DR},j}(V_j, n_j) = g_{\text{K-DR},j} n_j (V_j - V_K) \]
\[ I_{\text{Ca},j}(V_j, s_j) = g_{\text{Ca},j} s_j^2 (V_j - V_{\text{Ca}}) \]

where $j \in \{S, D\}$, $V_S$ and $V_D$ are the membrane potentials for the somatic and dendritic compartments respectively. Sodium has an activation variable, $m$, and an inactivation variable, $h$. Since sodium activates almost instantaneously compared to the other variables, its activation variable is represented by the steady state value of $m$, $m_\infty$, which is voltage dependent.

The activation for K-C and K-AHP ion channels depends on the calcium concentration inside the cell. Following standard practice [9], we model only the intracellular calcium concentration in a “shell” beneath the cell membrane, given by the variables $[Ca^{2+}]_S$ and $[Ca^{2+}]_D$. 
The variation of each concentration is described by a simple production/degradation model:

$$\frac{d[Ca^{2+}]_j}{dt} = -\phi I_{Ca,j} - \beta_j [Ca^{2+}]_j, \quad j \in \{S, D\}$$

where $\phi$ is the scaling constant (with arbitrary units) that converts the inward calcium current to the internal calcium concentration. We then have

$$I_{K-AHP,j}(V_j, q_j) = g_{K-AHP,j} q_j (V_j - V_K)$$

and

$$I_{K-C,j}(V_j, [Ca^{2+}]_j, c_j) = g_{K-C,j} c_j \min(1, [Ca^{2+}]_j / 250) (V_j - V_K)$$

where $j \in \{S, D\}$.

The gating variables $h_j, n_j, s_j, c_j, q_j, j \in \{S, D\}$, are each governed by an equation of the form

$$\frac{dy_j}{dt} = \frac{y\infty(U) - y_j}{\tau_y(U)} \text{ with } U = \begin{cases} V_j & \text{for } y_j \neq q_j \\ [Ca^{2+}]_j & \text{for } y_j = q_j \end{cases}$$

The associated steady state value and time constant are defined in the usual manner

$$y\infty(U) = \frac{\alpha_y(U)}{\alpha_y(U) + \beta_y(U)} \quad \text{and} \quad \tau_y(U) = \frac{1}{\alpha_y(U) + \beta_y(U)},$$

with rate constants as given in [9]:

$$\alpha_m(V_j) = \frac{0.32 \times (13.1 - V_j)}{e^{(13.1 - V_j)/4} - 1}, \quad \beta_m(V_j) = \frac{0.28 \times (V_j - 40.1)}{e^{(V_j - 40.1)/5} - 1}$$

$$\alpha_h(V_j) = 0.128 \times e^{(17 - V_j)/18}, \quad \beta_h(V_j) = \frac{4}{e^{(40 - V_j)/5} + 1}$$

$$\alpha_n(V_j) = \frac{0.016 \times (35.1 - V_j)}{e^{(35.1 - V_j)/5} - 1}, \quad \beta_n(V_j) = 0.25 \times e^{(0.5 - 0.025V_j)}$$

$$\alpha_s(V_j) = \frac{1.6}{1 + e^{-0.072 \times (V_j - 65)}}, \quad \beta_s(V_j) = \frac{0.02 \times (V_j - 51.1)}{e^{(V_j - 51.1)/5} - 1}$$
\[ \alpha_c(V_j) = \begin{cases} 2 \times e^{(6.5-V_j)/27} & \text{if } V_j > 50 \\ \left(e^{((V_j-10)/11)-(6.5)/27}\right)/18.975 & \text{otherwise} \end{cases} \]

\[ \beta_c(V_j) = \begin{cases} 0 & \text{if } V_j > 50 \\ 2 \times e^{(6.5-V_j)/27} - \alpha_c(V_j) & \text{otherwise} \end{cases} \]

\[ \alpha_q([Ca^{2+}]_j) = \min(0.00002 \times [Ca^{2+}]_j, 0.01) \]

\[ \beta_q = 0.001 \]

The synaptic current through AMPA receptor channels is given by \( I_{AMPA} \). Using Kirchoff’s Law, we obtain an expression for the voltage change across the membrane of each compartment:

\[ \frac{dV_S}{dt} = \frac{1}{C_m} \left\{ -I_{leak,S}(V_S) - I_{Na,S}(V_S, h_S) - I_{K-DR,S}(V_S, n_S) - I_{Ca,S}(V_S, s_S) - I_{K-C,S}(V_S, [Ca^{2+}]_S, c_S) - I_{K-AHP,S}(V_S, q_S) + \frac{g_e}{p} \cdot (V_D - V_S) + \frac{I_s}{p} \right\} \]

\[ \frac{dV_D}{dt} = \frac{1}{C_m} \left\{ -I_{leak,D}(V_D) - I_{K-AHP,D}(V_D, q_D) - I_{Ca,D}(V_D, s_D) - \frac{I_{AMPA}}{1-p} - I_{K-C,D}(V_D, [Ca^{2+}]_D, c_D) + \frac{g_e}{1-p} \cdot (V_S - V_D) + \frac{I_D}{1-p} \right\} \]

The maximal ionic conductances were chosen to be consistent with those in Traub’s multi-compartment model. In particular, the densities in our somatic compartment are consistent with the soma and proximal dendrite compartments of Traub’s model and those in our dendritic compartment are consistent with those in the distal dendrite compartments of Traub’s model. The values used are shown in Table 1. All other parameter values, set as in [9], are given in Table 2.

The model was implemented in Windows using XPPAUT, a differential equation simulation tool developed by B. Ermentrout. See [3] for details. The Runge-Kutta fourth-order explicit method was used with a fixed timestep of 0.05 ms.
2.1 Single cell model results By injecting a sustained current into our somatic compartment ($I_S$) and later into our dendritic compartment ($I_D$), we reproduced behaviour found experimentally and obtained with the 19-compartment model of [9]. When a sustained current is applied to the somatic compartment, a steady train of somatic action potentials is produced with no bursting (Figure 2(a), cf. with Figure 12 of [9]). The amplitude and frequency of both the somatic and dendritic voltage spikes are very similar to those obtained with Traub et al.’s 19 compartment model, which are consistent with experimentally observed values [8]. The interspike interval (i.e., the interval between action potentials) lengthens over time, which is consistent with experimental findings for CA1 neurons [4, 7, 8]. When a current of the same strength is applied to the dendritic compartment, our model accurately simulates the dendritic calcium spike and somatic burst observed experimentally [12]. The single burst is followed by a series of action potentials as seen in the simulations of Traub’s 19 compartment model. These results are shown in Figure 2(b) (cf. Figure 12 of [9]). The ac-
oration potentials occur at a lower frequency than when the same current is applied to the somatic compartment. This is due to the increased calcium-dependent outward potassium channel ($I_{K-\text{AHP,D}}$) triggered by the large dendritic calcium spike, resulting in a longer afterhyperpolarization. For the simulations in Figure 2, we use a constant applied current of either $I_S$ or $I_D = 1.25 \mu A/cm^2$ (which translates to approximately 0.48 nA in Traub’s model), while the current in the other compartment is held constant at $-0.25 \mu A/cm^2$. We have chosen this value so that it is slightly below the rheobase—the minimum amount of current required to excite the neuron—which is found to be approximately $-0.175 \mu A/cm^2$ when $p = 0.5$. We did so in accordance with [9], who applied a small negative current to suppress spontaneous firing. The coupling parameters are kept for now at the standard values of $g_C = 1.5 mS/cm^2$ and $p = 0.5$. Keeping all other parameters fixed, spiking behaviour with physiologically reasonable frequencies was observed for $1.25 \mu A/cm^2 \leq I_S < 3.5 \mu A/cm^2$ and bursting behaviour was observed for $0.5 \mu A/cm^2 \leq I_D < 4 \mu A/cm^2$.

As with Pinsky and Rinzel’s [6] CA3 model, our model only demonstrates the desired characteristic CA1 behaviour for a limited range of the coupling conductance, $g_C$. A very large coupling conductance essentially makes our model a single compartment model, and a very small $g_C$ leaves us with isolated compartments. However, if $g_C$ is slightly reduced then no bursting occurs, and if $g_C$ is slightly increased then aperiodic bursting occurs. Using an applied current of $I_D = 1.25 \mu A/cm^2$, the desired bursting behaviour is reproduced for $1.35 mS/cm^2 \leq g_C \leq 1.7 mS/cm^2$.

2.2 Bursting mechanics When a moderate sustained current is applied to the dendritic compartment, a somatic transient burst results (Figure 3(a)). To explain this burst qualitatively, we examine a single isolated model CA1 neuron with no synaptic input. Keeping the coupling parameters at their usual values of $g_C = 1.5 mS/cm^2$ and $p = 0.5$, and maintaining $I_S = -0.25 \mu A/cm^2$, a constant applied dendritic current of $I_D = 1.25 \mu A/cm^2$ is used to create the initial burst followed by the action potential shown in Figure 3(a). We find that the interplay between the outward dendritic currents $I_{K-C,D}$ and $I_{K-\text{AHP,D}}$ with the inward dendritic current $I_{Ca,D}$ is the primary cause of bursting. Since the calcium-dependent potassium currents are essential to the bursting behaviour, we look at the associated variables $[Ca^{2+}]_S$, $[Ca^{2+}]_D$, $q_S$ and $q_D$. In addition, the coupling parameters $g_C$ and $p$ are identified as key elements in generating this characteristic behaviour.
A sustained somatic current of $I_S = 1.25 \, \mu A/cm^2$ is applied

A sustained dendritic current of $I_D = 1.25 \, \mu A/cm^2$ is applied

FIGURE 2: The membrane voltage produced by the model when (a) a somatic and (b) a dendritic current is applied. Characteristic qualities of a CA1 neuron are reproduced: (a) a train of action potentials with increasing interspike interval, (b) a burst with a full dendritic calcium spike, followed by repetitive spiking.

To determine why our model is much more likely to produce a single transient burst than repetitive bursting, we consider the dynamics of the outward potassium currents $I_{K-C,D}$ and $I_{K-AHP,D}$. Recall that the activation of the K-C ion channels depends both on the fast voltage-dependent gating variables, $c_S$ and $c_D$, and on the slower intracellular calcium concentrations $[Ca^{2+}]_S$ and $[Ca^{2+}]_D$. Therefore the time constant of this outward potassium channel largely depends on the slow decay time ($1/\beta_{Ca^{2+}}$) of the concentration of intracellular calcium. Similarly, the decay time of the potassium current $I_{K-AHP,D}$ depends on the
slow gating variable $q_D$, which is also a function of $[Ca^{2+}]_D$. Thus the
dendritic calcium spikes repolarize in accordance with the time constants of
the slow variables $[Ca^{2+}]_S$, $[Ca^{2+}]_D$, $q_S$ and $q_D$. During the burst,
the length of the “quiet period” of $V_S$—when the soma is overdriven—is
also determined mainly by these slow variables. From Figure 3(b) it is
evident that the levels of $q_S$ and $q_D$ are low at the time of bursting, and
are significantly elevated during the action potentials. High $q$ values
permit greater outward flow of $I_{K-AHP}$, allowing the voltage to recover
faster, thereby reducing the probability of bursting. Similarly, the in-
tracellular calcium concentration is low during the initiation of a burst
(Figure 3(a)). An increased level of $[Ca^{2+}]$ activates the K-C channels
directly (and K-AHP channels indirectly), permitting the compartmen-
tal voltage to repolarize faster. This faster recovery implies a decreased
chance of bursting, resulting in single action potentials (Figure 3(a)). In
contrast, a large dendritic calcium conductance, $g_{Ca,D}$, creates a large
inward calcium current flow. This makes it difficult for the outward
potassium currents to counteract the inward flow, increasing the possi-
bility of a burst.

The coupling parameters, $g_C$ and $p$, play an essential role in gener-
ating bursts. Increased coupling results in increased flow of electrotonic
current, elevating the $q$ and $[Ca^{2+}]$ levels, as well as the inward current
flow. Although the model will be more effective in creating the bursts,
it will also recover faster, shortening the length of the burst. Thus, if
the coupling parameter $g_C$ is increased, the greater current flow between
the two compartments will be more effective at initiating a burst. Simi-
larly, the effects of the two compartments depend on their proportional
size, $p$: the larger the dendritic compartment is compared to the somatic
compartment, the more influence it will have.

3 Coupled cell model When excited, the presynaptic neuron re-
leases neurotransmitters into the synaptic cleft to signal its postsynaptic
neighbour. According to Pinsky and Rinzel [6] and Traub et al. [10],
the AMPA synapse is the dominant mechanism involved in the synchro-
nization of two pyramidal neurons. Using this knowledge, we construct a
standard AMPA synapse model [2] which synchronizes coupled cell mod-
els under a strong connection, and desynchronizes the coupled models
immediately once the strong connection is removed.

Since we have multiple cell models, we attach a subscript to our vari-
ables to denote the appropriate cell to which it belongs. Consider two
cells, a presynaptic cell (cell 2) and a postsynaptic cell (cell 1). Using the
FIGURE 3: (a) The somatic and dendritic intracellular calcium concentrations, $[Ca^{2+}]_S$ and $[Ca^{2+}]_D$ (in grey), overlaying the bursting and spiking. (b) $q_S$ and $q_D$ for the burst and spike sequence in (a). Therefore an applied dendritic current of $I_D = 1.25 \ \mu A/cm^2$ results in the same voltage burst and spike. It is evident that increased levels of the slow variables $[Ca^{2+}]_S$, $[Ca^{2+}]_D$, $q_S$, and $q_D$ decrease bursting activity. The characteristic transient burst occurs because of the initially low $q$ values.

gating variable $W_1$ and the maximal conductance $g_{AMPA_1}$, we represent the synaptic current due to the AMPA receptors by

$$I_{AMPA_1} = g_{AMPA_1}W_1(V_{D_1} - V_{EXC}),$$

where $V_{D_1}$ is the voltage of the dendritic compartment of the postsynaptic cell, and the reversal potential of the excitatory synapse is given
by $V_{EXC} = 60 \text{ mV}$ in accordance with Traub et al. [9]. The presynaptic voltage is denoted $V_{S_2}$ and $V_{D_2}$ for somatic and dendritic voltage respectively. During a signal (or spike) from the presynaptic cell, the presynaptic voltage increases. If $V_{S_2}$ surpasses some threshold, $V_W$, then the AMPA current of the postsynaptic cell activates with a time constant of $1 \text{ ms}$. Otherwise, the current will not activate. The activation degrades with a time constant $\tau_W$. This is modelled via the following equation

$$\frac{dW_1}{dt} = H(V_{S_2} - V_W) \frac{W_1}{\tau_W}$$

where $H(x)$ is the Heaviside function: $H(x) = 1$ if $x \geq 0$, $H(x) = 0$ otherwise. We set $V_W = 40 \text{ mV}$ and $\tau_W = 2 \text{ ms}$ in accordance with Traub [10] and Nadkarni and Jung [5]. To maintain a biologically realistic model, the two individual cell models should not be identical. Thus, we alter some variables of “cell 2” slightly, while keeping them close to the values for cell 1.

3.1 Coupled cell results Traub et al. [9] stated that the AMPA synapse is primarily responsible for the synchronization of two neurons, and that when AMPA blockers are applied, the neurons are reported to desynchronize rapidly. To demonstrate that this characteristic is upheld in our model, we couple two cell models with an AMPA synapse and apply a constant input to the dendritic compartment of each cell. Cell 1 receives a stronger dendritic input than cell 2 and hence undergoes a longer burst and subsequently spikes at a faster rate than cell 2. With a weak AMPA connection the cells do not synchronize, but with a strong enough AMPA connection, they do. These results are illustrated in Figure 4

Similar results are obtained when the cells receive a sinusoidal input to the somatic compartment. We set the frequency and strength of the input to be different for both cell 1 and cell 2, so the cells do not synchronize on their own. If the neuron models are connected with a sufficiently strong AMPA conductance, then the cell with the weaker input will synchronize to the cell with the stronger input. We show an example of these results in Figure 5, where the cells are connected strongly ($g_{AMP A_1} = g_{AMP A_2} = 0.2 \text{ mS/cm}^2$) until $t = 500 \text{ ms}$, and weakly ($g_{AMP A_1} = g_{AMP A_2} = 0.01 \text{ mS/cm}^2$) after $t = 500 \text{ ms}$. We set the input to be $I_{S_{cell_1}}(t) = I_{S_1} \sin(2\pi t/100) + 1$ so $I_{S_{cell_1}}(t)$ fluctuates between $-0.25$ and $2.25$. Similarly we set $I_{S_{cell_2}}(t) = I_{S_2} \sin(2\pi t/130) + 1$, so $I_{S_{cell_2}}(t)$ has a slightly different period.
FIGURE 4: Cell 1 receives dendritic input $I_{D_1} = 2.0 \mu A/cm^2$, cell 2 receives dendritic input $I_{D_2} = 1.25 \mu A/cm^2$. (a) A weak AMPA connection: $g_{AMPA_1} = 0.04 mS/cm^2$. (b) A strong AMPA connection: $g_{AMPA_2} = 0.2 mS/cm^2$.

4 Discussion Due to their susceptibility to excessive bursting and the organized nature of the hippocampal CA1 region, CA1 pyramidal neurons have been a focus of research involving seizure activity for many years [8, 9]. Biophysical models of these neurons exist [8, 9], but for large networks or the analysis of key characteristics, a simpler model may be required. Thus, we created a two-compartment model of hippocampal CA1 neurons. The model reproduces qualitatively and quantitatively much of the characteristic behaviour of a CA1 neuron, as identified by [9]. That is, when the soma or proximal dendrites are stimulated with a current less than 1 nA ($I_S < 1 nA$), a train of action potentials is exhibited. However, if the same current stimulates the distal den-
FIGURE 5: (a) The voltage of cell 1 and its corresponding input, $I_{S_{\text{cell}1}}(t)$. Note the synchronization to its input. (b) The voltage of cell 2 and the input from cell 1, $I_{S_{\text{cell}1}}(t)$. Cell 2 is synchronized to cell 1 until $t = 500 \text{ ms}$, at which point it desynchronizes. At $t = 500 \text{ ms}$ the connection goes from strong to weak, and the applied sinusoidal current to cell 2 dominates.

drites ($I_D < 1 \text{ nA}$), a full dendritic calcium spike with somatic burst is produced, followed by low frequency action potentials. The bursting mechanisms were analyzed in detail, and the slow variables $q$ and $[Ca^{2+}]$ were shown to influence the generation of a burst and the length of the interspike interval. It is clear from our analysis that a two compartment model is the minimal conductance based model which can reproduce the appropriate bursting behaviour. The benefit of our simplified model is that key parameters involved in characteristic behaviour can be analyzed, and since it maintains its biological relevance, predictions about potential behavioural responses can be made. In addition, computa-
tional efficiency ensures that large networks can be created easily. As a first step toward such network models, we have implemented an AMPA synapse model and used it to investigate synchronization of two coupled CA1 neurons.

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