OPTIMAL STRATEGY FOR CYCLIC STEAM STIMULATION OIL PRODUCTION: A MATHEMATICAL MODEL

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ABSTRACT. A central issue of managing a heavy oil production using cyclic steam stimulation process is whether an optimal strategy for adding new wells and removing old wells can be found. In this paper we present a mathematical model which provides a general framework for this type of problem. With suitable assumptions, we formulate the production process as a constrained optimal control problem in continuous time or equivalently a constrained linear programming problem in discrete time. The main advantage of the model is that the size of the problem is not necessarily related to the scale of the operations (number of wells). A numerical algorithm based on the model shown to be efficient is given, as well as results for two test cases of large scale projects.

1. Introduction. Cyclic steam stimulation process is commonly used to produce heavy oil from oil-sand formations. High pressure steam is generated at central plant facilities. The steam is distributed through a pipeline system and injected into reservoirs away from the central plant facilities. Steam injection continues until the oil viscosity is reduced to a level such that the oil can be pumped to the surface. The oil, water and gas mixture is produced during the production part of the cycle and is returned to the central plant facilities. Water is separated from the mixture, treated, mixed with make-up water and reused as feed water for the steam generators. The produced gas supplements the purchased natural gas as fuel for the steam generators. The produced oil is processed by removing sand and diluted with a lighter hydrocarbon (diluent) to meet pipeline viscosity specifications. The diluted bitumen is sold to refineries. For details of the process, readers are referred to [2].

Typically central plant facilities are built in several locations, surrounded by wells drilled at various stages of the operation. Each well is associated with a single plant. The older wells may have completed up to 10 cycles of steaming and production. New wells are periodically added. The duration of the steaming and production phases depends
on the age of the well. The steaming phase may last from 28 to more than 150 days. The production cycle may last from 100 to 1600 days. Old wells can only be abandoned at the end of their production cycle.

An important issue related to the cyclic steam stimulation operation is whether the performance of the overall system can be optimized. Given the vast scale of the operation with thousands of wells and the complicated interdependencies of the system, an ad hoc approach of abandoning and drilling wells may not be the most desirable method. It is natural to seek a more sophisticated and systematic strategy which optimizes at least some aspects of the operation.

In this paper we formulate the process of adding and removing wells during the operation as a constrained optimal control problem in continuous time with two unknown functions and a constrained linear programming (LP) problem in discrete time. In the continuous time case, the number of wells satisfies a first order differential equation. The rate of removing wells is an unknown control function of time and age of the wells. The number of new wells appears as an unknown boundary condition. There are many candidates for the objective function. The overall profit is chosen in this study, subject to constraints on production level and steam consumption. To formulate the process as an LP problem, we can discretize the continuous formulation by assuming that decisions are made at the beginning of a finite number of intervals and wells can be put into discrete age groups. The primary variable is the number of wells in each age group at the beginning of each time interval. Constraints are imposed on this primary unknown variable while we search the optimal solution to maximize the profit.

A major advantage of our formulation is that the objective function and constraints are linear functionals of the control variables in which the size of the problem is related to the number of classes of the wells, not the number of wells. As a result, the solution of the discrete case can be obtained using a standard LP algorithm for large scale systems. Numerical results using an LP algorithm are presented for two test cases, with 1200 wells existing at the beginning and another 6000 new wells to be added during the operation of the system.

The organization of the rest of the paper is as follows. In Section 2 a detailed description of a typical operation is given. In Section 3 we introduce the continuous linear optimal control model, followed by the
discrete LP formulation in Section 4. Numerical examples with one and two classes of wells are given in Section 5. We finish the paper with a brief discussion in Section 6.

2. Problem description. We start by making the following observations and assumptions for an otherwise complex oil-field using the cyclic steam stimulation process.

- The oil production and steam consumption of each well is not known in general. Accurate prediction requires detailed modeling using fluid dynamics and statistical tools [1, 8, 9, 10]. However, on the scale of a realistic operation, it is not unreasonable to model the production and steam consumption by a simple mathematical formula, based on the data from field operation. Typically, production rate is high at the early stage and decreases as the wells become old.

- The actual cost of developing and maintaining wells during production cycles often varies, depending on many factors. For simplicity, we will use an empirical formula fitted from the field operation data.

- We recognize that wells are in general different. We assume that they can be categorized into different classes: from ‘good’ wells with high production level to ‘bad’ wells with low production.

- The steam is normally distributed from several plants to wells at various locations through the pipelines. A main physical constraint is the daily available steam. In reality, the steam available to wells also differs from one location to another. However, for simplicity, we will assume that the location factor can be ignored and the only constraint is the total daily available steam.

- The oil recovered from the wells (with water, sand and gas) is ‘cleaned up,’ diluted, transported and sold. The water is treated and recycled back to the steam plants to make up for part of the water supply needed for generating the steam. There is a limit on the amount of water which can be used at a given time. However, this constraint is completely ignored here.

- The price of the crude oil is obviously volatile. However, we will ignore the stochastic nature of the oil price and treat it as a constant.

- Another physical constraint is that there is an upper bound on the daily oil production due to the availability of transportation and
treatments. Although the constraint on each individual well is location-dependent, we will only impose a constraint on the total production level.

- Economic consideration provides a lower bound on the production level, which likely becomes important only towards the end of the entire operation. For simplicity, this constraint is ignored in this study.

We observe that, unless each well is completely different, it is not necessary to distinguish individual wells. Thus, we can put the wells into a number of classes and introduce the averaged production, cost and steam requirement for each class as follows. First of all, we have the averaged daily production function

$$P(a, k) = 18e^{-0.0003a}g(k)$$

m$^3$/day, where $a$ is the age of the well (measured in days since the beginning of the operation), $g(k)$ is the class function to be defined and $k$ is the class index. Note that the production is averaged over the production and steam cycles of the well. In reality, a well produces oil only when it is in the production cycle. The averaged daily steam rate (assumed to be class independent) can be expressed as a function of age as

$$S(a) = 115(a + 1)^{-0.18}$$

m$^3$/day. Again, in reality, the steam is only required during the steam cycles. The averaged cost of maintaining normal production of a well is also assumed to be class independent and is given as

$$C(a) = 601(a + 1)^{-0.105}$$

in dollars. All three are monotonically decreasing functions. The cost of developing a new well is assumed to be a constant, $D_0$. In addition, we assume that the discount rate $R$ is a constant. With these assumptions, we are now in the position of formulating the problem.

3. Mathematical model. It is worth pointing out that several approaches can be used to formulate the problem. For example, we can use the starting and ending times of each individual well as primary variables. We can also use the number of wells and the age of wells
as primary variables. When the wells can be put into a small number of classes, the advantage of the latter approach is that the number of wells is not relevant to the size of the problem. Therefore, it is more efficient for a system with a large number of wells, which is our main interest here.

Define \( n(t, a, k) \) as the number of class \( k \) wells at time \( t \) and \( a \) as the age of the well. \( n_0(t, k) = n(t, 0, k) \) is used to describe the number of wells being developed at time \( t \). Let \( n_b(a, k) = n(0, a, k) \) denote the number of wells at the beginning of the operation (the starting profile). The total number of class \( k \) wells \( N(k) = \int_0^{T_a} n_b(a, k) \, da + \int_0^{T_\infty} n_0(t, k) \, dt \) is given. \( T_a \) and \( T_\infty \) are the natural life span of wells and the time of the entire operation of the field, respectively. The developing cost of a well is given as \( D(a) = D_0 \delta(a+) \).

The total profit is a functional \( J(n) \) defined as

\[
J(n) = \sum_{k=1}^K \int_0^{T_\infty} \int_0^{T_a} n(t, a, k) F(a, k) \exp(-Rt) \, da \, dt,
\]

where

\[
F(a, k) = p_0 P(a, k) - C(a) - D(a)
\]

is the profit of operating one class \( k \), age \( a \) well before discount and \( p_0 \) is the crude oil price. The objective is to find function \( n(t, a, k) \) such that \( J(n) \) is maximized subject to the following constraints

\[
\sum_{k=1}^K \int_0^{T_a} n(t, a, k) S(a) \, da \leq S^H,
\]

\[
\sum_{k=1}^K \int_0^{T_a} n(t, a, k) P(a, k) \, da \leq P^H,
\]

where \( S^H \) is the upper steam rate limit, \( P^H \) is the upper production level.

We now discuss the dynamic constraint satisfied by \( n(t, a, k) \). We note that the total number of wells is a conserved quantity except when they are shut down or removed from operation. Define \( f(t, a, k) \geq 0 \) as the
number of class $k$ wells of age $a$ removed during a unit time interval at time $t$. Then we must have
\begin{equation}
\frac{\partial n}{\partial t} + \frac{\partial n}{\partial a} = -f(t, a, k).
\end{equation}

This is a first order hyperbolic equation and it can be easily solved symbolically using the method of characteristics
\begin{align*}
n(t, a, k) &= n(0, a - t, k) - \int_{a-t}^{a+t} f\left(\frac{\eta - a + t}{2}, \frac{\eta + a - t}{2}, k\right) d\eta \\
n(t, a, k) &= n(t - a, 0, k) - \int_{t-a}^{t+a} f\left(\frac{\eta - a + t}{2}, \frac{\eta + a - t}{2}, k\right) d\eta
\end{align*}
for $a > t$ and

for $t \geq a$. Note that $n(t - a, 0, k) = n_0(t - a, k)$ is unknown and $n(0, a - t, k) = n_b(a - t, k)$ is given. It can be observed as well that $n(t, a, k)$ is a nonincreasing function and the inequalities
\begin{align*}
n(t, a, k) &< n_0(t - a, k) \quad \text{or} \quad n(t, a, k) < n_b(a - t, k)
\end{align*}
hold when the wells are being removed along the characteristics, $\xi = a - t = \text{constants}$. Otherwise, $n(t, a, k)$ is simply $n_0(t - a, k)$ or $n_b(a - t, k)$.

To summarize, the model we propose here falls into a class of optimal control problems governed by partial differential equations (PDEs) \[15\]. The objective functional is determined by an unknown variable $n$ which itself satisfies a PDE with unknown control functions $f(t, a, k)$, the rate of removing wells, and $n_0(t, k)$, the number of new wells developed at time $t$. Optimal control problems have been an active research subject and readers are referred to references \[4\] and \[7\] for more details. The main difficulty of finding an exact solution of our problem is due to the state constraints. Obviously, without state constraints, the problem is trivial and the solution is simply $n(t, a, k) = n_b(a - t, k)$ or $n(t, a, k) = n_0(t - a, k)$ with $f(t, a, k) = 0$ and $n_0(t, k) = N(k)\delta(t)$ when $T_\infty$ is sufficiently large.

4. Discretization. One approach for finding the extremum of a functional is to use direct methods. For example, one can use the Ritz
method to search for an optimization sequence [3]. We now discuss a special case of the method where the integrals are approximated by a numerical quadrature.

Suppose that a grid with \((t_r, a_s)\) is set up to cover the solution domain \([0, T\infty] \times [0, T_a]\), with \(t_r = t_{r-1} + T\), \(a_s = a_{s-1} + T\) for \(r = 1, 2, \ldots, I_t\) and \(s = 1, 2, \ldots, I_a\). The functional \(J(n)\) can be rewritten as

\[
J(n) = \sum_{k=1}^{K} \sum_{r=1}^{I_t} \sum_{s=1}^{I_a} \int_{t_{r-1}}^{t_r} \int_{a_{s-1}}^{a_s} n(t, a, k) F(a, k) \exp(-Rt) \, da \, dt.
\]

We now discuss how to approximate the integrals on the intervals between the grid points. We assume that the decisions to add new wells and remove old wells are made at the beginning of each time interval \([t_{r-1}, t_r]\) and the smallest unit in time is one day. Define a well of stage \(s\) as between \(a_{s-1}\) and \(a_s - 1\) days old. Thus, there are \(I_a\) number of stages, with \(T_a\) being the oldest age of the wells.

We approximate the profit functional (8) as

\[
J(n) = \sum_{k=1}^{K} \sum_{r=1}^{I_t} \sum_{s=1}^{I_a} v(r, s, k) z(r, s, k),
\]

where

\[
v(r, s, k) = \sum_{j=0}^{T_s-1} \left\{ P[a_{s-1} + j, k] p_0 - C(a_{s-1} + j) \right\} \cdot \exp(-R(t_{r-1} + j)) - D(s) \exp(-Rt_{r-1})
\]

is the Riemann sum of the gross income (production times price) that a class \(k\) well of stage \(s\) will generate over a \(T\) day interval minus the operating cost of a well of stage \(s\), all discounted. Note that \(\delta t = \delta a = 1\) as time and age are measured in days. The variable

\[z(r, s, k) = n(t_{r-1}, a_{s-1}, k)\]

is the number of class \(k\) wells of stage \(s\) in the \(r\)th decision interval. We define \(z(0, s, k)\) to be the starting profile of the field. Note that the function \(v(r, s, k)\) is nonlinear, but for any fixed pair \((r, s)\) it is simply a constant. Thus, equation (9) is linear.
The cost of developing a well is
\begin{align}
D(s) = \begin{cases} 
D_0 & \text{if } s = 1 \\
0 & \text{otherwise},
\end{cases}
\end{align}
which only needs to be paid when a well is first developed.

The steam constraint can be approximated as
\begin{align}
\sum_{k=1}^{K} \sum_{s=1}^{I_t} S(a_{s-1}) z(r, s, k) \leq S^H
\end{align}
for \( r = 1, \ldots, I_t \). Note that \( S \) is decreasing and thus the constraint only needs to be checked at grid points. Moreover, \( S(a_{s-1}) \) is a constant for a fixed \( s \) and thus equation (12) defines \( I_t \) linear constraints. Similarly, we now have \( I_t \) linear constraints for the upper bound on the production rate for all \( r \),
\begin{align}
\sum_{k=1}^{K} \sum_{s=1}^{I_t} P(a_{s-1}, k) z(r, s, k) \leq P^H.
\end{align}
The dynamic or transition constraints can be written as
\begin{align}
z(r + 1, s + 1, k) \leq z(r, s, k),
\end{align}
since all wells of stage \( s \) must either be abandoned or become wells of stage \( s + 1 \) the next decision interval. If \( a_s \) is not an abandon time, then equation (14) will be a strict equality. Finally, to ensure we do not develop more wells than are available, we must have
\begin{align}
\sum_{r=1}^{I_t} z(r, 1, k) \leq N(k),
\end{align}
where \( N(k) \) is the total number of class \( k \) wells in the field, with \( \sum_k N(k) = N \), and \( N \) is the total number of wells. Finding the solution to (9)–(15) is a constrained integer programming problem.

Note that for large problems, we drop the integer declaration on \( n \) or \( z \) and solve it as an LP problem for real numbers, rounding the answers down to the nearest integer. If the total number of wells is
large enough, then this solution should be close to the optimal integer solution. Indeed, this procedure relies on the fact that we are dealing with a large number of wells and the results will be less accurate for small scale operations.

Obviously, other types of discretization can be used. For example, instead of using a grid function, we can use piecewise polynomials on each subinterval. From a numerical point of view, using grid functions fits into a finite difference framework while using a piecewise polynomial approximation is a finite element method. From the LP point of view, using grid functions can be interpreted as restrict adding/removing wells at specific times and using piecewise polynomials means that we will drill/abandon wells continuously with some restrictions.

Other related issues are the convergence of the discrete solution to the solution of the continuous problem (if it exists) in general, and the speed of convergence associated with a particular discretization as \( T \) decreases. However, addressing these issues properly is beyond the scope of this paper. Instead, we now present the results obtained using this special discretization.

5. **Numerical solution.** To solve the discrete LP problem, we wrote a program that takes an input file which defines a problem and writes it as an LP-file, which was submitted to an LP-solver [6]. The whole process takes about 30 minutes on a P-II based PC.

In our examples \( T_\infty = 11000 \) days and \( T = 100 \) days. We assume that \( g(1) = 1 \) for the one-class problem and \( g(1) = 0.8 \) and \( g(2) = 1.2 \) for the ‘bad’ and ‘good’ wells of the two-class problem. Other parameter values are \( D_0 = 500,000, p_0 = 200, S^H = 120,000 \) and \( P^H = 42,000 \). For the one-class problem, we have \( I_t = T_\infty / T = 110 \) decision intervals over the life of the project. The oldest age of wells is assumed to be \( T_a = 7000 \) days. Hence, there are \( I_a = T_a / T = 70 \) different stages of well ages. Thus, we have a linear objective function with \( I_t \) linear constraints for the steam requirements, \( I_t \) linear constraints for each of the production bounds, and a total of 76,931 linear transition constraints. The total number of decision variables is 76,930. The solution procedure is illustrated via a simpler example problem in the Appendix. For the two-class problem, the size increases but the basic feature of the problem remains unchanged. The starting profile for the
TABLE 1. The starting profile.

<table>
<thead>
<tr>
<th>Age of Well</th>
<th>100</th>
<th>300</th>
<th>500</th>
<th>1000</th>
<th>3000</th>
<th>5500</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-Class:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Starting number of wells</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Two-Class:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Starting number of “good” wells</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Starting number of “bad” wells</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

project is given in Table 1. The production cycle end times are at \( \{300, 500, 800, 1100, 1600, 2300, 3200, 4300, 5600, 7000\} \) days. Recall that wells can only be abandoned at the end of each production cycle.

In Figure 1, the solution of the number of wells as a function of time and age intervals is plotted for the one-class problem. Note that there are 110 time intervals and 70 age intervals which consist of 100 days in our computation. For ease of discussion, we will refer to them simply as

FIGURE 1. 3D plot of \( n(t, a, 1) \) as a function for \( t \) and \( a \) for the one-class case.
time and age in the following. The value on the time axis is the number of new wells being developed during the operation and the value on the age axis is the number of existing wells at the beginning of the operation. It can be seen that these values are being propagated along the characteristics, indicated by the traveling wave form along the lines $t - a = \text{constants}$, until being set to zero (abandoned). Most of the new wells are added at the beginning and again at time step 43, when old wells are abandoned (at the age of 4300 days). This is indicated by two peaks on the time axis and the discontinuity of the wave form of the first peak.

For the two-class case, the solution for the number of good and bad wells is plotted in Figure 2. Similar traveling wave forms can be seen clearly from the figure. A peak on the time axis at time zero in Figure 2a suggests that most of the good wells are developed at the beginning of the operation. They are then abandoned at age 5600 days when most of the bad wells are developed (time step 56) shown by a peak on the time axis in Figure 2b.

For a more detailed examination of the solution, Figure 3 shows cross-sections of Figure 1 at 1000 day intervals (up to 5000 days). These plots show the profile of the oil field (the number of wells producing of each age category) at particular times in the future. The title of the plot indicates the future time displayed in that plot. The horizontal-axis is the age of the wells in the project and the vertical-axis is the number of wells of each age. It can be seen that the day 100 plot is almost identical to the starting profile we specified. The day 1000 through day 4000 plots show the effect of the starting profile. Most of the wells that exist at the start are still active in 1000 days, but nearly all of the starting wells have been abandoned by day 4000. The day 5000 plot shows the middle evolution of the project when the wells developed at time zero are abandoned and a second wave of new wells was added until we exhausted our supply of undeveloped wells.

The cross-sections for the two-class well case are presented in Figures 4 and 5, good wells represented by the figures on the left and the bad ones represented by figures on the right. From these plots, it can be seen that good wells are developed at the beginning of the operation, indicated by a big peak in numbers of wells of age zero, and a second wave of good wells during the next 4000 days. On the contrary, most of the bad wells are developed after around day 4000.
FIGURE 2. 3d plot of $n(t, a, k)$ as a function of $t$ and $a$ for the two-class case: (a) $k = 1$ for the good wells; and (b) $k = 2$ for the bad wells.
for the following 4000 days. Therefore, the optimal strategy, based on these plots, is to develop the good wells first and the bad wells are only drilled afterwards. Note that without the constraints, the optimal strategy will be to develop all the wells at the beginning. Therefore, the strategy adapted here is mostly influenced by the constraints.

Next we look at the constraints. Figures 6 and 7 shows the daily production rate over the life of the project and the daily steam requirements over the life of the project. We can see that with this particular input file the operation is restricted by production constraints at the initial period. The steam constraints become the main restricting mechanism afterwards. In spite of some differences in details, the one-class and two-class cases behave in a similar fashion.
FIGURE 4. Snapshots of the field profile on days 100 through 2000 with good wells on the left and the bad wells on the right.

Plotted in Figures 8 and 9 are the costs and profits over the life of the project. These plots show the daily profits and costs and do not include the one-time costs associated with drilling new wells. The magnitude of those development costs can be estimated from the number of new wells developed during the operation.

A summary of some interesting statistics is presented in Tables 2 and 3. Not included in the tables are the values of the objection function, or the total profit of the project. For the one-class case, the total profit is approximately $3.0057 \times 10^{10}$. For the two-class project, the profit is slightly higher, approximately $3.1050 \times 10^{10}$. 
FIGURE 5. Snapshots of the field profile on days 3000 through 5000 with good wells on the left and the bad wells on the right.

TABLE 2. The values of characteristics for the project with one class of wells.

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active Wells</td>
<td>3794</td>
<td>1872</td>
<td>4252</td>
</tr>
<tr>
<td>Daily Production</td>
<td>34145</td>
<td>12989</td>
<td>42000</td>
</tr>
<tr>
<td>Daily Steam Use</td>
<td>106381</td>
<td>57319</td>
<td>106381</td>
</tr>
<tr>
<td>Daily Costs</td>
<td>956849</td>
<td>574415</td>
<td>1107894</td>
</tr>
<tr>
<td>Daily Profits</td>
<td>5056171</td>
<td>896901</td>
<td>7489346</td>
</tr>
</tbody>
</table>
FIGURE 6. The daily production as a function of time: thin line for the one-class case and the thick line for the two-class case.

FIGURE 7. The daily required steam as a function of time: thin line for the one-class case and the thick line for the two-class case.
FIGURE 8. The daily cost as a function of time: thin line for the one-class case and the thick line for the two-class case.

FIGURE 9. The daily profit as a function of time: thin line for the one-class case and the thick line for the two-class case.
TABLE 3. The values of characteristics for the project with two classes of wells.

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active Good Wells</td>
<td>1980</td>
<td>0</td>
<td>3400</td>
</tr>
<tr>
<td>Active Bad Wells</td>
<td>1567</td>
<td>500</td>
<td>2500</td>
</tr>
<tr>
<td>Daily Production</td>
<td>30641</td>
<td>7557</td>
<td>42000</td>
</tr>
<tr>
<td>Daily Steam Use</td>
<td>107760</td>
<td>59609</td>
<td>120000</td>
</tr>
<tr>
<td>Daily Costs</td>
<td>970305</td>
<td>590915</td>
<td>1113474</td>
</tr>
<tr>
<td>Daily Profits</td>
<td>5157829</td>
<td>920398</td>
<td>7555749</td>
</tr>
</tbody>
</table>

Before we finish this section, we note that adding more constraints should not be a problem as long as they are linear. As we have demonstrated, the complexity of the problem increases when more classes of wells are considered. If we have \( K \) classes of wells there will be \( K \) times as many decision variables and \( K \) times as many transition constraints.

6. Conclusion. In this paper we have presented a mathematical model and an optimal control/linear programming formulation for adding and removing wells in a heavy oil production project using a cyclic steam stimulation process. The major advantage of our model is that the complexity of the solution does not depend on the number of wells that are being modeled if the wells can be put into a small number of groups (classes). Thus, the approach is more suitable for large scale projects involving thousands of wells. Using two test cases, we have demonstrated that solutions for projects with realistic scales can be obtained efficiently.

The main purpose of this paper is to provide a general framework for finding optimal operation strategy of cyclic steam stimulation oil production and similar problems. We have made several major assumptions to simplify the problem. For example, we have assumed that the characteristics of the wells are known. We have also assumed constant values for the interest rate and for the price of the commodity produced. In reality, these are stochastic in nature. An interesting and relevant complication to the problem would be to investigate the optimal strategy when some of the characteristics or the price/interest rate
become stochastic. A possible approach to this problem will be to use the techniques of stochastic programming. Such an investigation will be a subject of a future study.

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Appendix

7.1 List of symbols.

\begin{itemize}
  \item $a$ Age of the well (in days)
  \item $k$ Class index
  \item $t$ Time since the beginning of operation (in days)
  \item $g(k)$ Class function
  \item $P(a,k)$ Average daily production
  \item $S(a)$ Average daily steam rate
  \item $C(a)$ Averaged cost of production
  \item $D_0$ Cost of developing a well
  \item $R$ Discount rate
  \item $n(t,a,k)$ Number of wells at time $t$ of age $a$ of class $k$
  \item $n_0(t,k)$ Number of wells of class $k$ being developed at time $t$
  \item $n_0(a,k)$ Number of wells of class $k$ at the beginning of
    development (the starting profile)
  \item $N(k)$ The total number of wells of class $k$
  \item $T_a$ Maximum lifespan of a well
\end{itemize}
T_\infty \quad \text{Lifespan of the project}

D(A) \quad \text{Development cost of well}

J(n) \quad \text{Total profit functional}

F(a,k) \quad \text{Profit of operating one well of class } k \text{ of age } a

p_0 \quad \text{Crude oil price}

S^H \quad \text{Upper daily total steam rate limit}

P^H \quad \text{Upper daily total production limit}

f(t,a,k) \quad \text{Number of class } k \text{ wells of age } a \text{ removed at time } t

T \quad \text{Length of a decision interval (discrete model)}

t_r \quad \text{Decision points (discrete model)}

a_s \quad \text{Well ages at decision points (discrete model)}

I_n \quad \text{Number of well stages (discrete model)}

s \quad \text{The stage of well (discrete model)}

r \quad \text{A decision interval (discrete model)}

v(r,s,k) \quad \text{Gross income from a class } k \text{ well of stage } s \text{ over the decision interval } r

z(r,s,k) \quad \text{The number of class } k \text{ wells of stage } s \text{ in the } r\text{th decision interval}

N \quad \text{The total number of wells}

7.2 \textbf{A sample problem for the LP approach.} To solidify the idea of using the LP approach, we look at a simpler problem’s input and output files. In what follows, t_is = z(i,j) in the notation of Section 4.

\textbf{Input file:}

\textbf{Number of undeveloped wells:}

3000

\textbf{Time step (days):}

100

\textbf{Total time length (days):}
Abandon times (days):
100  300  400
Starting well ages (days):
100  200  300
Number of wells starting with those ages:
400  400  400
Max steam use (day):
60000
Max production rate (day):
20000
Min production rate (day):
9000
Discount rate (annual):
0.05
Price of commodity:
200
Well development cost:
0

LP file:

/* Objective Function */
max:  311216 t1s1 + 306425 t1s2 + 298215 t1s3 + 289585 t1s4
+ 306981 t2s1 + 302256 t2s2 + 294157 t2s3 + 285645 t2s4
+ 302805 t3s1 + 298144 t3s2 + 290155 t3s3 + 281758 t3s4;

/* Steam Constraints */
115 t1s1 + 50.11 t1s2 + 44.27 t1s3 + 41.17 t1s4 <= 60000;
115 t2s1 + 50.11 t2s2 + 44.27 t2s3 + 41.17 t2s4 \leqslant 60000; \\
115 t3s1 + 50.11 t3s2 + 44.27 t3s3 + 41.17 t3s4 \leqslant 60000;

/* Lower production bound constraints */
17.99 t1s1 + 17.46 t1s2 + 16.95 t1s3 + 16.45 t1s4 \geq 9000; \\
17.99 t2s1 + 17.46 t2s2 + 16.95 t2s3 + 16.45 t2s4 \geq 9000; \\
17.99 t3s1 + 17.46 t3s2 + 16.95 t3s3 + 16.45 t3s4 \geq 9000;

/* Upper production bound constraints */
17.99 t1s1 + 17.46 t1s2 + 16.95 t1s3 + 16.45 t1s4 \leq 20000; \\
17.99 t2s1 + 17.46 t2s2 + 16.95 t2s3 + 16.45 t2s4 \leq 20000; \\
17.99 t3s1 + 17.46 t3s2 + 16.95 t3s3 + 16.45 t3s4 \leq 20000;

/* Transition Constraints */
t1s1 + t2s1 + t3s1 \leq 3000; \\
t1s2 \leq 400; t1s3 = 400; t1s4 \leq 400; \\
t2s2 \leq t1s1; t2s3 = t1s2; t2s4 \leq t1s3; \\
t3s2 \leq t2s1; t3s3 = t2s2; t3s4 \leq t2s3;

/* Declarations */
int t1s1, t1s2, t1s3, t1s4, t2s1, t2s2, t2s3, t2s4, \\
t3s1, t3s2, t3s3, t3s4;

Solution File:

Value of objective function: 942033225

t1s1 = 95, t1s2 = 400, t1s3 = 400, t1s4 = 275,
$$t_{2s1} = 183, \ t_{2s2} = 95, \ t_{2s3} = 400, \ t_{2s4} = 400,$$

$$t_{3s1} = 262, \ t_{3s2} = 183, \ t_{3s3} = 95, \ t_{3s4} = 400$$

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