REFLECTION OF WAVES IN TRANSVERSELY ISOTROPIC MICROPOLAR THERMOELASTIC HALF-SPACE

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ABSTRACT. In the present article, we discuss the problem of reflection of plane wave incident at the surface of transversely isotropic micropolar thermoelastic medium without energy dissipation. The wave equations are solved by imposing proper conditions on displacements, stresses and temperature distribution. It is found that there exist four different waves viz., quasi-longitudinal displacement (qLD) wave, quasi transverse displacement (qTD) wave, quasi transverse microrotational (qTM) wave and quasi thermal wave (qT). Amplitude ratio of these reflected waves are presented, when different waves are incident. Numerically simulated results have been depicted graphically for different angle of incidence with respect to frequency. Some special cases of interest also have been deduced from the present investigation.

1 Introduction The propagation of waves in thermoelastic materials has many applications in various fields of science and technology, namely, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessel, aerospace, chemical pipes and metallurgy. The importance of thermal stresses in causing structural damages and changes in functioning of the structure is well recognized whenever thermal stress environments are involved. Therefore, the ability to predict electrodynamics stress induced by sudden thermal loading in composite structures is essential for the proper and safe design and the knowledge of its response during the service in these severe thermal environments.

Material is endowed with microstructure, like atoms and molecules at microscopic scale, grains and fibers or particulate at mesoscopic scale. Homogenization of a basically heterogeneous material depends on scale of interest. When stress fluctuation is small enough compared to mi-
crostructure of material, homogenization can be made without considering the detailed microstructure of the material. However, if it is not the case, the microstructure of material must be considered properly in a homogenized formulation \cite{2, 6}. The concept of microcontinuum, proposed by Eringen \cite{2}, can take into account the microstructure of material while the theory itself remains still in a continuum formulation. The first grade microcontinuum consists a hierarchy of theories, such as, micropolar, microstretch and micromorphic, depending on how much micro-degrees of freedom is incorporated. These microcontinuum theories are believed to be potential tools to characterize the behavior of material with complicated microstructures.

The most popular microcontinuum theory is micropolar one, in this theory, a material point can still be considered as infinitely small, however, there are microstructures inside of this point. So there are two sets of variable to describe the deformation of this material point, one characterizes the motion of the inertia center of this material point; the other describes the motion of the microstructure inside of this point. In micropolar theory, the motion of the microstructure is supposed to be an independently rigid rotation. Application of this theory can be found in \cite{2, 7}.

Recently, the theory of thermoelasticity without energy dissipation, which provides sufficient basic modifications to the constitutive equation to permit the treatment of a much wider class of flow problems, has been proposed by Green and Naghdi \cite{5} (called the GN theory). The discussion presented in the above reference includes the derivation of a complete set of governing equations of the linearized version of the theory for homogeneous and isotropic materials in terms of displacement and temperature fields and a proof of the uniqueness of the solution of the corresponding initial mixed boundary value problem. Chandrasekharullah and Srinath \cite{1} investigated one-dimensional wave propagation in the context of the GN theory.

The aim of the paper is to study the reflection of waves in transversely isotropic micropolar thermoelastic medium without energy dissipation. The propagation of waves in micropolar materials has many applications in various fields of science and technology, namely, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessel, aerospace, chemical pipes and metallurgy. The graphical representation is given for amplitude ratios of various reflected waves for different incident waves at different angle of incidence, i.e., for $\theta = 30^\circ, 45^\circ, 90^\circ$. 


2 Basic equations  The basic equations in dynamic theory of the plain strain of a homogeneous, transversely isotropic micropolar medium following Eringen [3] and Green and Naghdi [5] in the theory of thermoelasticity of without energy dissipation in absence of body forces, body couples and heat sources are given by

\begin{align}
  t_{ji,j} &= \rho \ddot{u}_i, \\
  m_{ik,i} + \varepsilon_{kmn}t_{mn} &= \rho j \ddot{\phi}_k,
\end{align}

and heat conduction equation is given by

\begin{equation}
  \kappa_{ij}T_{ij} = \rho C^\theta \frac{\partial^2 T}{\partial t^2} + T_0 \frac{\partial^2}{\partial t^2} \beta_{ij}u_{i,j}.
\end{equation}

The constitutive relations can be given as

\begin{align}
  t_{ij} &= A_{ijkl}\varepsilon_{kl} + G_{ijkl}\Psi_{kl} - \beta_{ij}T, \\
  m_{ij} &= B_{ijkl}\varepsilon_{kl} + G_{klji}\Psi_{kl},
\end{align}

where

\begin{equation}
  \varepsilon_{ij} = u_{j,i} + \epsilon_{jik}\phi_k, \quad \Psi_{ij} = \phi_{i,j}.
\end{equation}

In these relations, we have used the following notations: \( \rho \) is the density, \( \varepsilon_{ijk} \) is the permutation symbol, \( u_i \) is the components of displacement vector, \( \phi_k \) is the component of microrotation vector, \( t_{ij} \) is the components of the stress tensor, \( m_{ij} \) is the components of the couple stress tensor, \( \varepsilon_{ij} \) is the components of micropolar strain tensor, \( \kappa_{ij} \) are the characteristic constants of the theory, and \( \beta_{ij} = A_{ijkl}\alpha_{kld} \) are the thermal elastic coupling tensor.

3 Formulation of the problem  We consider homogeneous, transversely isotropic micropolar medium under the theory of thermoelasticity without energy dissipation initially in an undeformed state and at uniform temperature \( T_0 \). We take the origin of coordinate system on the plane surface and \( x_3 \) axis pointing normally into the half-space, which is thus represented by \( x_3 \geq 0 \). We consider plane waves in plane such that all particles on a line parallel to \( x_2 \)-axis are equally displaced. Therefore, all the field quantities will be independent of \( x_2 \) coordinate. So, we...
assume the components of the displacement and microrotation vector of the form

\[ \vec{U} = (u_1, 0, u_3), \quad \vec{\phi} = (0, \phi_2, 0). \]

With the aid of equation (6), equations (1)–(4) reduce to

\[ \begin{align*}
A_{11} \frac{\partial^2 u_1}{\partial x_1^2} + (A_{13} + A_{56}) \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + A_{55} \frac{\partial^2 u_1}{\partial x_3^2} \\
+ K_1 \frac{\partial \phi_2}{\partial x_3} - \beta_1 \frac{\partial T}{\partial x_1} &= \rho \frac{\partial^2 u_1}{\partial t^2}, \\
A_{66} \frac{\partial^2 u_3}{\partial x_1^2} + (A_{13} + A_{56}) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + A_{33} \frac{\partial^2 u_3}{\partial x_3^2} \\
+ K_2 \frac{\partial \phi_2}{\partial x_1} - \beta_3 \frac{\partial T}{\partial x_3} &= \rho \frac{\partial^2 u_3}{\partial t^2}, \\
B_{77} \frac{\partial^2 \phi_2}{\partial x_1^2} + B_{66} \frac{\partial^2 \phi_2}{\partial x_3^2} - X \phi_2 + K_1 \frac{\partial u_1}{\partial x_3} + K_2 \frac{\partial u_3}{\partial x_1} &= \rho \frac{\partial^2 \phi_2}{\partial t^2}, \\
\kappa_1 \frac{\partial^2 T}{\partial x_1^2} + \kappa_3 \frac{\partial^2 T}{\partial x_3^2} &= \rho C^* \frac{\partial^2 T}{\partial t^2} + T_0 \frac{\partial^2}{\partial t^2} \left( \beta_1 \frac{\partial u_1}{\partial x_1} + \beta_3 \frac{\partial u_3}{\partial x_3} \right),
\end{align*} \]

where

\[ \begin{align*}
\beta_1 &= A_{11} \alpha_1 + A_{13} \alpha_3, \\
\beta_3 &= A_{31} \alpha_1 + A_{33} \alpha_3, \\
K_1 &= A_{56} - A_{55}, \\
K_2 &= A_{66} - A_{56}, \\
X &= K_2 - K_1.
\end{align*} \]

For simplification we use the following nondimensional variables:

\[ \begin{align*}
x'_i &= \frac{x_i}{L}, \quad u'_i = \frac{u_i}{L}, \quad m'_{ij} = \frac{m_{ij}}{L^2 T_0}, \quad \phi'_i = \frac{\rho c^2}{\beta_1 T_0} \phi_i, \\
T' &= \frac{T}{T_0}, \quad c^2 = \frac{A_{11}}{\rho},
\end{align*} \]

where \( L \) is a parameter having dimensions of length and \( c_1 \) is the longitudinal wave velocity of the medium.

**4 Solution of the problem** Let \( \vec{p}'(p_1, 0, p_3) \) denote the unit propagation vector, \( c \) and \( k \) are respectively the phase velocity and the wave number of the plane waves propagating in \( x_1x_3 \)-plane.
We seek plane wave solution of the equations of motion of the form

\[ (u_1, u_3, \phi_2, T) = (\bar{u}_1, \bar{u}_3, \bar{\phi}_2, \bar{T}) e^{i(\xi x_1 + \eta x_3 - ct)}. \]

With the help of equations (11) and (12) in equations (7)–(10), we get four homogeneous equations in four unknowns. Solving the resultant system of equation for nontrivial solution, we obtain

\[ Ae^\delta + Bc^\delta + Cc^4 + Dc^2 + E = 0, \]

where

\[
\begin{align*}
A &= b_4 - \frac{b_5}{\omega^2}, \quad B = g_1 + \frac{g_6}{\omega^2}, \quad C = g_3 + \frac{g_2 + g_4}{\omega^2} + \frac{g_4}{\omega^4}, \\
D &= g_5 + \frac{g_7}{\omega^2}, \quad E = g_8, \quad g_1 = b_1 - b'_2 - b_4a_1, \\
g_2 &= b_8 - b_2 - b_4a_1 + b_1a_1 + b_2a_15 - a_1b_5, \\
g_3 &= b'_6 - b_1 - b_3 + b'_4a_1 + b'_1a_10, \\
g_4 &= b_6 + a_1b_2 + a_10(b_12 - b'_13) + 14(b_15 - b_18 + b'_15) - a_1b_8, \\
g_5 &= b_7 + b_2a_1 + b_9a_10 - a_1b'_6, \quad g_6 = b_5a_1 - a_15b_22, \\
g_7 &= b_4a_1 + b_20a_15 - b_6a_1 - a_10b_10, \quad g_8 = -b_7a_1 - b_11a_10, \\
g_9 &= b_9a_15 - a_1b_8 - a_10b_13, \quad b_1 = d_7a_8d_12, \\
b_2 &= d_7(a_8d_11 + d_9a_6) + a_7a_4d_12, \quad b'_2 = a_2a_9d_12, \\
b_3 &= a_8(a_6d_7 + a_2d_12), \quad b_4 = d_12d_7a_9, \quad b_5 = d_7d_11a_9, \\
b_6 &= a_0a_7a_4 + a_8(a_2d_11 - a_3a_5), \quad b'_6 = a_2a_6a_9, \quad b_7 = a_2a_6a_8, \\
b_8 &= a_9(a_2a_11 + a_3a_5), \quad b_9 = a_8a_11d_12, \\
b_{10} &= a_11(a_8d_11 + a_6d_9) + a_5a_8a_12 - a_6a_7a_13, \quad b_{11} = a_8a_6a_11, \\
b_{12} &= -a_7a_13d_12, \quad b'_{12} = a_9a_{11}d_12, \quad b_{13} = a_7a_{13}d_11, \\
b'_{13} &= a_9(a_11d_12 + a_3a_12), \quad b_{14} = a_8(a_1a_3 + a_2a_12), \\
b_{15} &= a_7(a_4a_12 - a_3a_13), \quad b'_{15} = a_9(a_1a_3 + a_2a_12), \quad b_{16} = a_8a_{12}d_7, \\
b_{17} &= -a_9a_{12}d_7, \quad b_{18} = d_{12}(a_2a_11 + a_2a_{13}), \\
b_{19} &= d_{11}(a_{14}a_4 + a_2a_13) - a_6a_{13}d_7 + a_5(a_{14}a_11 - a_3a_{13}), \\
b_{20} &= a_6(a_4a_{11} + a_2a_{13}), \quad b_{21} = a_{13}d_{12}d_7, \quad b_{22} = a_{13}d_7d_{11},
\end{align*}
\]
The roots of this equation gives four values of \( c^2 \). Four positive values of \( c \) will be the velocities of propagation of four possible waves. The waves with velocities \( c_1, c_2, c_3 \) and \( c_4 \) correspond to four types of quasi waves. Let us name these waves as quasi-longitudinal displacement (qLD) wave, quasi transverse displacement (qTD) wave, quasi transverse microrotational (qTM) wave and quasi thermal wave (qT).

5 Reflection of waves We consider a transversely isotropic micropolar thermoelastic half-space without energy dissipation occupying the region \( x_3 \geq 0 \). Incident qLD or qTD or qTM or qT wave at the interface will generate reflected qLD, qTD, qTM and qT waves in the half space \( x_3 > 0 \). The total displacements, microrotation and temperature distribution are given by

\[
(u_1, u_3, \phi_2, T) = \sum_{j=1}^{8} A_j (1, r_j, s_j, t_j) e^{iB_j},
\]

where

\[
B_j = \omega \left[ t - \frac{x_1 \sin e_j - x_3 \cos e_j}{c_j} \right], \quad j = 1, 2, 3, 4,
\]

\[
B_j = \omega \left[ t - \frac{x_1 \sin e_j + x_3 \cos e_j}{c_j} \right], \quad j = 5, 6, 7, 8,
\]
\( \omega \) is the angular frequency. Here subscripts 1, 2, 3, 4, respectively, denote the quantities corresponding to incident qLD, qTD, qTM and qT wave whereas the subscripts 5, 6, 7 and 8, respectively, denote the corresponding reflected waves. Substituting the values for \( p_1 \) and \( p_3 \):

- for incident qLD wave: \( p_1 = \sin e_1, \quad p_3 = -\cos e_1 \);
- for incident qTD wave: \( p_1 = \sin e_2, \quad p_3 = -\cos e_2 \);
- for incident qTM wave: \( p_1 = \sin e_3, \quad p_3 = -\cos e_3 \);
- for incident qT wave: \( p_1 = \sin e_4, \quad p_3 = -\cos e_4 \);
- for reflected qLD wave: \( p_1 = \sin e_5, \quad p_3 = \cos e_5 \);
- for reflected qTD wave: \( p_1 = \sin e_6, \quad p_3 = \cos e_6 \);
- for reflected qTM wave: \( p_1 = \sin e_7, \quad p_3 = \cos e_7 \);
- for reflected qT wave: \( p_1 = \sin e_8, \quad p_3 = \cos e_8 \).

Here \( e_1 = e_5, \quad e_2 = e_6, \quad e_3 = e_7 \) and \( e_4 = e_8 \), i.e., the angle of incidence is equal to the angle of reflection in transversely isotropic micropolar generalized thermoelastic medium, so that the velocities of reflected waves are equal to their corresponding to their corresponding incident waves, i.e., \( c_1 = c_5, \quad c_2 = c_6, \quad c_3 = c_7 \) and \( c_4 = c_8 \).

### 6 Boundary condition

The boundary conditions are given by

\[
\begin{align*}
    t_{33} &= 0, \quad t_{31} = 0, \quad m_{32} = 0, \quad \frac{\partial T}{\partial x_3} + hT = 0.
\end{align*}
\]

where \( h \) is the surface heat transfer coefficient; \( h \rightarrow 0 \) corresponds to thermally insulated boundaries and \( h \rightarrow \infty \) refers to isothermal boundaries.

\[
\begin{align*}
    t_{33} &= \frac{\partial u_1}{\partial x_1} + \frac{1}{d_7} \frac{\partial u_3}{\partial x_3} - d_{18}T, \\
    t_{31} &= d_{16} \frac{\partial u_3}{\partial x_1} + d_{17} \phi_2 + \frac{\partial u_1}{\partial x_3}, \\
    m_{32} &= d_{15} \frac{\partial \phi_2}{\partial x_3}.
\end{align*}
\]

The wave numbers \( k_j, \ j = 1, 2, \ldots, 8 \) and the apparent velocity \( c_j, \ j = 1, 2, \ldots, 8 \) are connected by the relation

\[
\begin{align*}
    k_1 e_1 = k_2 e_2 = \cdots = k_8 e_8 = \omega
\end{align*}
\]
at the surface $x_3 = 0$. Relation (18) may also be written in order to satisfy the boundary conditions (16) as

\begin{equation}
\frac{\sin e_1}{c_1} = \frac{\sin e_2}{c_2} = \cdots = \frac{\sin e_8}{c_8} = \frac{1}{c}.
\end{equation}

Making use of equations (14), (17) and (19) in the boundary conditions (16) (thermally insulated boundaries), we obtain

\begin{equation}
\sum_{j=1}^{8} A_{ij}A_j = 0, \quad i = 1, \ldots, 4,
\end{equation}

where

\begin{align*}
A_{1j} &= \frac{\sin e_j}{c_j} + \frac{r_j \cos e_j}{c_j} - t_j d_{18j}, & j &= 1, \ldots, 4, \\
A_{2j} &= d_1 \cos e_j c_j + d_{16} r_j \frac{\sin e_j}{c_j} + d_{17} s_j, & j &= 1, \ldots, 4, \\
A_{1j} &= \frac{\sin e_j}{c_j} - \frac{r_j \cos e_j}{c_j} - t_j d_{18j}, & j &= 5, \ldots, 8, \\
A_{2j} &= -d_1 \cos e_{j} c_j + d_{16} r_j \frac{\sin e_j}{c_j} + d_{17} s_j, & j &= 5, \ldots, 8, \\
A_{3j} &= d_{15} s_j \frac{\cos e_j}{c_j}, & A_{4j} &= t_j \frac{\cos e_j}{c_j}, & j &= 1, \ldots, 4, \\
A_{3j} &= -d_{15} s_j \frac{\cos e_j}{c_j}, & A_{4j} &= -t_j \frac{\cos e_j}{c_j}, & j &= 5, \ldots, 8.
\end{align*}

**Incident qLD wave**: In case of incident qLD wave, $A_2 = A_3 = A_4 = 0$. Dividing set of equations (20) throughout by $A_1$, we obtain a system of four nonhomogeneous equations in four unknowns which can be solved by Gauss elimination method and we have

\begin{equation}
Z_i = \frac{A_{i+4}}{A_1} = \frac{\Delta^1_i}{\Delta}, \quad i = 1, \ldots, 6.
\end{equation}

**Incident qTD wave**: In case of incident qTD wave, $A_1 = A_3 = A_4 = 0$ and thus we have

\begin{equation}
Z_i = \frac{A_{i+4}}{A_2} = \frac{\Delta^2_i}{\Delta}, \quad i = 1, \ldots, 6.
\end{equation}
Incident qTM wave: In case of incident qTM wave, \( A_1 = A_2 = A_4 = 0 \) and thus we have

\[
Z_i = \frac{A_{i+4}}{A_3} = \frac{\Delta^3_i}{\Delta}, \quad i = 1, \ldots, 6
\]

Incident qT wave: In case of incident qT wave, \( A_1 = A_2 = A_3 = 0 \) and thus we have

\[
Z_i = \frac{A_{i+4}}{A_1} = \frac{\Delta^4_i}{\Delta}, \quad i = 1, \ldots, 6
\]

where

\[
\Delta = |A_{i+4}|_{4 \times 4},
\]

and \( \Delta^p_i \) \((i = 1, 2, 3, 4) \) (\( p = 1, 2, 3, 4 \)) can be obtained by replacing, respectively, the 1st, 2nd, \ldots, 4th column of \( \Delta \) by \([-A_1, -A_2, -A_3, -A_4]^T\).

7 Numerical results and discussion  
In order to illustrate the theoretical results obtained in the preceding sections, we now present some numerical results. For numerical computation, we take the values for relevant parameters for micropolar transversely isotropic thermoe- 

dastic solid as

\[
A_{11} = 21.4 \times 10^9 N/m^2, \quad A_{77} = 5.4 \times 10^9 N/m^2,
\]

\[
A_{88} = 5.2 \times 10^9 N/m^2, \quad A_{22} = 20.24 \times 10^9 N/m^2,
\]

\[
A_{12} = 9.4 \times 10^9 N/m^2, \quad A_{78} = 4.0 \times 10^9 N/m^2,
\]

\[
B_{44} = .779 \times 10^5 N, \quad B_{66} = .779 \times 10^5 N.
\]

Following Gauthier \[4\] we take, the nondimensional values for Alu-


mium epoxy like composite as

\[
\rho = 2.19 \times 10^3 kg/m^3, \quad \lambda = 9.4 \times 10^9 N/m^2, \quad \mu = 4.0 \times 10^9 N/m^2,
\]

\[
K = 1.0 \times 10^9 N/m^2, \quad C^* = 1.04 Cal/K, \quad \gamma = 0.779 \times 10^5 N,
\]

\[
j = 0.2 \times 10^{-4} m^2.
\]

Graphical representation is given for the variations of amplitude ra-

tios of reflected qLD, qTD, qTM and qT waves when four types of waves
viz. qLD, qTD, qTM and qT waves are incident at the free surface to compare the results in two cases, one for the waves incident from micropolar transversely isotropic without energy dissipation (MTIWOED) and other from micropolar isotropic isotropic without energy dissipation (MIWOED). In Figures 1–4, the graphical representation is given for variations of amplitude ratios $|Z_1|$, $|Z_2|$, $|Z_3|$ and $|Z_4|$ in case of incident qLD wave. Figures 5–8, 9–12 and 13–16, respectively, show the same situation in case of incident qTD, qTM, and qT waves. Here $|Z_1|$, $|Z_2|$, $|Z_3|$ and $|Z_4|$ are, respectively, the amplitude ratios of reflected qLD, qTD, qTM and qT wave. These variations are shown for three different angle of incidence viz., $\theta = 30^\circ$, $45^\circ$, $90^\circ$. In these figures the solid lines corresponds to the case of MTIWOED while the dotted lines corresponds to the case of MIWOED. Also, the solid lines without center symbol, lines with center symbol ($-\circ-\circ-$), solid lines with center symbol ($-\times-\times-$), respectively, represent variations for $\theta = 30^\circ$, $\theta = 45^\circ$ and $\theta = 90^\circ$ in case on MTIWOED, whereas the corresponding broken lines represent represent the same condition in the case of MIWOED.

**Incident qLD wave:** It is observed from Figure 1 that the amplitude ratio $|Z_1|$ of reflected qLD wave first increases sharply to peak value at an angle for $\omega = 5$, then sharply decreases and ultimately increases to become constant at the end, when $\theta_0 = 45^\circ$ for MTIWOED and MIWOED. While, for all the other cases, its value initially increases and then oscillate to become constant ultimately.

Figure 2 indicate the variations of amplitude ratio $|Z_2|$ of reflected qTD wave which shows that in the case of MTIWOED, the value of $|Z_2|$ initially oscillate, then steadily increases with increase in frequency, for all the three angles of inclination. However, for MIWOED and initial inclination, its value oscillate with very large amplitude. As the angle of inclination increases, the oscillatory behaviour disappears and its value become steady.

It is depicted from Figures 3 and 4 that the values of amplitude ratio $|Z_3|$ and $|Z_4|$, for all the angles and for both MTIWOED and MIWOED, goes on increasing with increase in frequency. The difference depicted in the figures is that, for $|Z_3|$, the value goes on decreasing with increase in angle of inclination, while for $|Z_4|$ its value start with initial oscillation of very small amplitude and afterwards shows the reverse behaviour.

**Incident qTD wave:** The variations of amplitude ratios of various reflected waves for incident qTD wave is shown in Figures 5–8. The amplitude ratio $|Z_1|$ sharply decreases to become constant for all the
FIGURE 1: Amplitude ratio $|Z_1|$ when qLD wave is incident.

FIGURE 2: Amplitude ratio $|Z_2|$ when qLD wave is incident.
FIGURE 3: Amplitude ratio $|Z_3|$ when qLD wave is incident.

FIGURE 4: Amplitude ratio $|Z_4|$ when qLD wave is incident.
cases, with slight differences in their amplitudes.

It can be seen from Figure 6 that the value of amplitude ratio $|Z_2|$ for MTIWOED sharply decreases for $\omega = 7$, then slightly increases to attain a constant value. At $\psi = 45^\circ$, its value sharply increases, then decreases to become constant, while at $\psi = 90^\circ$ its value initially oscillate to become constant with increase in frequency. However, for MIWOED, the amplitude ratio shows the reverse behaviour with slight difference in their amplitudes.

Figures 7 and 8 show the variations of amplitude ratio $|Z_3|$ and $|Z_4|$ sharply decreases within the range $0 \leq \omega \leq 10$, then become constant with increase in frequency, for MTIWOED. However, for MIWOED, and all the angle of incidence, their values initially oscillates and then increases with increase in frequency.

**Incident qTM wave:** The variations of amplitude ratio $|Z_1|$ are almost shows an oscillating behavior attaining certain maxima and minima within the range $0 \leq \omega \leq 20$, but after this range it shows linear behaviour. Also, the values for MTIWOED are higher as compared to those for MIWOED. These variations are depicted in Figure 9.

For the amplitude ratio $|Z_2|$ (Figure 10) of reflected qTD wave, for initial angle of incidence, its value sharply increases, then sharply de-
FIGURE 6: Amplitude ratio $|Z_2|$ when qTM wave is incident.

FIGURE 7: Amplitude ratio $|Z_3|$ when qTM wave is incident.
creases to attain a constant value. With increase in angle of incidence, its value initially oscillates and attain a constant value. The variations for MIWOED is similar to those obtained in the case of MTIWOED, with opposite behaviour at initial angle of inclination.

Figure 11 shows that the curves for $|Z_3|$ start with difference in their pattern of variation but end with almost similar type of variation. It can be seen from this figure that the value of $|Z_3|$ shows the similar behaviour for initial angle of incidence, while with increase in angle of incidence, its value shows the opposite behaviour.

It is observed from Figure 12 that $|Z_4|$ initially oscillate then show a sudden decrease at $\theta = 30^\circ$, goes on increasing with increase in frequency at $\theta = 45^\circ$, while oscillate and then decreases to become constant at $\theta = 90^\circ$. The variations for the case of MIWOED, start with oscillating behaviour showing peaks at particular value of frequency and become constant at the end. It is observed that the values get increased with increase in angle of incidence.

**Incident qT wave:** The variations of amplitude ratios of various reflected waves for incident qT wave is shown in Figures 13–16. The amplitude ratio $|Z_4|$ sharply decreases to become constant for all the cases, with slight differences in their amplitudes.

FIGURE 8: Amplitude ratio $|Z_4|$ when qTM wave is incident.
FIGURE 9: Amplitude ratio $|Z_1|$ when qT wave is incident.

FIGURE 10: Amplitude ratio $|Z_2|$ when qT wave is incident.
FIGURE 11: Amplitude ratio $|Z_3|$ when $qT$ wave is incident.

FIGURE 12: Amplitude ratio $|Z_4|$ when $qT$ wave is incident.
The variations of $|Z_2|$ and $|Z_3|$ indicated in Figures 14 and 15. It can be seen from these figures that the values oscillate within the interval $(0, 20)$, showing the peaks of different amplitudes. After this interval the values for all the cases become steady.

Variations in the values of $|Z_4|$ indicate that anisotropy as well as angle of incidence show a significant impact on it throughout the whole range (Figure 16). The behavior of $|Z_4|$ oscillatory within the range $0 \leq \omega \leq 40$. The values of amplitude ratio $|Z_4|$ first increase from small values to maximum with small oscillations and ultimately decrease to become steady. The values for MTIWOED are higher as compared to those for MIWOED at $\varepsilon = 30^\circ, 45^\circ$, but the behavior is reversed with further increase in the angle of incidence. Anisotropy show a greater impact on $|Z_4|$ as compared to the angle of incidence.

FIGURE 13: Amplitude ratio $|Z_1|$ when qTD wave is incident.
FIGURE 14: Amplitude ratio $|Z_2|$ when qTD wave is incident.

FIGURE 15: Amplitude ratio $|Z_3|$ when qTD wave is incident.
8 Conclusion  The analytic expressions of amplitude ratios for various reflected waves are obtained for the micropolar transversely isotropic medium possessing thermoelastic properties without energy dissipation. It is concluded from the graphs that the values of amplitude ratios $|Z_1|$, $|Z_2|$ and $|Z_3|$ show sharp oscillations at initial frequencies for incident $q_{LD}$ and $q_{TD}$ waves, as compared to $q_{TM}$ and $q_{T}$ incident waves. An appreciable effect of anisotropy and angle of incidence is observed on amplitude ratios of various reflected waves.

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