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POLEWARD EXPANSION OF HADLEY CELLS

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ABSTRACT. Reanalyses of climate data for recent decades have indicated that the Hadley cells of the atmospheric circulation are expanding toward the poles as well as slowing in their circulation velocity. Similarly, recent meteorological data show a poleward movement of the jet streams that affect midlatitude weather. Although the precise mechanism of these changes in Hadley cells and jet streams is not fully understood, it is believed to be linked with global warming. In this paper we study a simple mathematical model to investigate whether such qualitative changes could be induced purely by the physics of rotating spherical convection. The model consists of the Navier-Stokes equations for a Boussinesq fluid, rotating in a spherical shell, differentially heated with a latitudinal surface temperature gradient. Many other factors that influence the atmospheric circulation are excluded from this model. A decrease in the pole-to-equator temperature gradient in the model leads to an expansion and slowing of the Hadley circulation as well as a poleward movement of jet streams, thus suggesting a simple mechanism for these observed climate changes.

1 Introduction The Intergovernmental Panel on Climate Change (IPCC), in its Fourth Assessment Report (AR4) [**25**], has presented strong evidence that the climate of the Earth has changed significantly in recent decades, and is likely to continue to do so, at least in part due to increases in anthropogenic greenhouse gas emissions. The IPCC has reported that the globally averaged temperature of the Earth is rising. The polar regions are warming most rapidly while the temperatures of the mid-latitudes and tropics are changing less dramatically. The change in albedo as polar ice melts yields positive feedback that accelerates the polar warming. An immediate consequence of this enhanced polar warming is a reduction in the pole-to-equator temperature gradient.

Two Hadley cells straddle the Earth's equator, extending approximately ±30 degrees latitude in each hemisphere, but shifting north–south with the seasons. They are the dominant feature of the general atmospheric circulation in the tropical latitudes and also have a significant influence in the subtropics [**2, 6**]. The two Hadley cells meet near the equator in the *intertropical convergence zone* (ITCZ), which is characterized by warm moist air rising through the troposphere, and abundant rainfall. At the poleward boundaries of the Hadley cells there are high pressure regions of descending dry air called *subtropical highs*. Most of the Earth's major deserts are found at these latitudes. On the surface, convection and Coriolis forces together produce the easterly *trade winds* that blow in each hemisphere from the subtropical highs to the ITCZ.

Ground-based and satellite instrumental data together provide evidence that the Hadley cells have expanded poleward by about 2 to 4.5 degrees of latitude in the past 27 years, and their circulation velocity is decreasing [**3, 7, 19, 20**]. GCM simulations forecast that this poleward expansion of the Hadley cells, together with a slowing of their circulation, will continue throughout this century [**16, 24**]. A poleward expansion of the Hadley cells implies a poleward movement of the subtropical highs that could lead to serious desertification of economically important regions such as the southwestern USA, southern Europe and Australia [**10, 18, 23**]. The details of the mechanisms causing this widening of the tropical belt are not yet fully understood. A comprehensive review of these issues by Seidel *et al.* [**24**] indicates that the observed widening is occurring even faster than predicted by the suite of IPCC models. Furthermore, Archer and Caldeira [**1**] analyzed the behaviour of the jet streams in meteorological datasets from 1979 to 2001 and found a poleward movement of the jet streams in both hemispheres.

A great many factors influence the circulation of the atmosphere. It is the purpose of this article to study a simple mathematical model of spherical convection, excluding all other factors, to determine whether physical convection in a rotating spherical geometry may be sufficient to reproduce at least qualitatively the changes described in the previous paragraph. This mathematical model consists of a rotating spherical shell of fluid with radial gravity and differential heating on the inner boundary, using the Navier-Stokes Boussinesq partial differential equations. In the solution of the model equations, both the extent and the velocity of the Hadley cell circulation vary strongly and nonlinearly with changes in the pole-to-equator surface temperature gradient. The model suggests that the same factors that are causing differential warming of the polar regions on Earth could indirectly cause the poleward expansion and weakening of the Hadley cells as well as the poleward shift of the jet streams, described in the previous paragraph. In addition, the behaviour of the model is consistent with the fact that the Hadley cells of Venus are much larger than those of Earth, as discussed in Section 3.

2 The mathematical model The model studied in this paper is based on the model presented in [**12**]. After [**12**] was submitted for publication, the authors became aware of the recent observational evidence of poleward expansion and slowing of the Hadley cells of the Earth reported in the climate literature [**7, 10, 16, 18, 19, 20, 23, 24, 25**]. However, the model in [**12**] was not designed to address such issues. Therefore, a change is made from a no-slip to a stress-free boundary condition on the outer boundary, in order to better model a planetary atmosphere at the tropopause, and the solutions of the modified PDE boundary value problem are recomputed for values of the parameters that are more relevant for large-scale flow. Furthermore, the model equations in this paper are transformed to nondimensional form to facilitate comparisons with such flows. The new results are compared to the changes in the Earth's atmospheric circulation reported in the climate change literature cited above, and lead to the discussion in Section 3.

Although the above changes are made to provide a closer resemblance to large-scale flow, the model is kept as simple as possible, incorporating only the fundamental physical principles that are essential for convection in a rotating spherical shell of fluid with radial gravity. The model is assumed rotationally symmetric (zonally invariant) because this is a study of variations of climate with latitude, not longitude. Therefore, important longitudinal phenomena such as the el Nino Southern Oscillation (ENSO) are missing in this model. Seasonal oscillations are assumed to be averaged out and we impose a north-south reflectional symmetry across the equator. Thus we need solve the resulting system of partial differential equations (given in (2) below) only in the northern hemisphere. The temperature gradient that drives convection in the model is imposed latitudinally on the inner boundary surface, and it is assumed that the surface temperature is determined by the solar heating of the surface. The variation with colatitude θ of the average annual flux of solar radiation on a planet with axis of rotation tilted approximately $20°$ with respect to the plane perpendicular to the solar rays is approximately proportional to $-\cos(2\theta)$; see [**11**]. Therefore, the boundary condition for the temperature on the inner boundary surface is chosen to be

$$T = T_r - \Delta T \cos(2\theta), \tag{1}$$

where θ is the colatitude (measured from the pole), T_r is the reference temperature and $2\Delta T$ is the total difference in temperature from pole to equator. On Earth, the temperature distribution at the surface is affected by many factors in addition to solar influx, such as differences in the surface albedo, greenhouse gases and the ocean transport of heat from the tropics to poles. In the model these factors are modeled collectively by changes in the parameters ΔT and T_r. It is expected that changes in the manner in which the differential heating is applied would not change the qualitative dynamics, as is the case for differentially heated rotating fluids in cylindrical annuli [**21**].

The remaining boundary conditions are constants. There is no heat flux (insulated boundary condition) at the outer boundary (tropopause), which is assumed to have constant height for convenience. The fluid velocity vector $\mathbf{u}$ satisfies the no-slip boundary condition at the inner surface and stress-free boundary condition at the outer surface. The symmetry assumptions impose natural boundary conditions at the pole and equator [**12**].

The atmosphere in this model is a Boussinesq fluid; that is, it satisfies the Navier-Stokes equations and has density that is independent of pressure but depends linearly on temperature. This is the simplest mathematical fluid that exhibits physically realistic convection. Boussinesq fluids are widely used in studies of convection, even though the Earth's atmosphere is not exactly Boussinesq [**5, 11**]. Heat is transported in this model only by convection and diffusion. Diffusion of heat in this model serves as a stand-in for a variety of thermal transport mechanisms found in the atmosphere, including radiative transfer and turbulence. The remarkable relevance of laboratory-scale experiments and numerical simulations to the climate of the Earth has been discussed for example in [**15,** Ch. 3] and [**21**].

The Navier-Stokes Boussinesq partial differential equations, in a coordinate system rotating with the spheres at angular velocity Ω, are

$$
\begin{aligned}
\frac{\partial \mathbf{u}}{\partial t} &= \nu \nabla^2 \mathbf{u} - 2\boldsymbol{\Omega} \times \mathbf{u} + [g\mathbf{e}_r + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\qquad \alpha\,(T - T_r) - \frac{1}{\rho_0}\nabla p - (\mathbf{u}\cdot\nabla)\,\mathbf{u} \\
\frac{\partial T}{\partial t} &= \kappa \nabla^2 T - (\mathbf{u}\cdot\nabla)\,T, \\
0 &= \nabla \cdot \mathbf{u}.
\end{aligned}
\tag{2}
$$

Here the fluid velocity vector is $\mathbf{u} = \mathbf{u}(r, \theta, t) = w\mathbf{e}_r + v\mathbf{e}_\theta + u\mathbf{e}_\varphi$, the pressure is $p = p(r, \theta, t)$, the temperature is $T = T(r, \theta, t)$, ν is the kinematic viscosity, $\mathbf{\Omega}$ is the rotation vector, α is the coefficient of thermal expansion, ρ_0 is the reference fluid density, g is the gravitational acceleration and κ is the thermal diffusivity. The two cross-product terms represent the Coriolis and centrifugal forces in the rotating coordinate system.

Next we make a rescaling to the nondimensional variables $\mathbf{u}'$, t', $\mathbf{r}'$, T' and p':

$$\mathbf{u} \to U\mathbf{u}', \qquad t \to \tau t', \qquad \mathbf{r} \to R\mathbf{r}', \qquad T \to \Delta T\, T', \qquad p \to Pp',$$

and choose

$$L = r_0, \qquad U = \Omega r_0, \qquad \tau = \frac{L}{U} = \frac{1}{\Omega}, \qquad P = \rho_0 R \Omega^2 r_0,$$

where R and r_0 are the gap width and inner radius, respectively, of the spherical shell; then drop the primes and obtain the equations in nondimensional form

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} &= 2E_k \nabla^2 \mathbf{u} - \nabla p - 2\widehat{\mathbf{\Omega}} \times \mathbf{u} \\ &\qquad + R_{oT}(\mathbf{e}_r - \eta^2 F \widehat{\mathbf{\Omega}} \times \widehat{\mathbf{\Omega}} \times \mathbf{r})(T - T_r) - \frac{1}{\eta}(\mathbf{u} \cdot \nabla)\mathbf{u} \\ \frac{\partial T}{\partial t} &= \frac{1}{Pe} \nabla^2 T - \frac{1}{\eta}(\mathbf{u} \cdot \nabla) T \\ 0 &= \nabla \cdot \mathbf{u}, \end{aligned} \tag{3}$$

where $\widehat{\mathbf{\Omega}}$ is a unit vector in the direction of the rotation vector, E_k is the Ekman number, R_{oT} is the thermal Rossby number (similar to a Burger number), F is a Froude number, Pe is the Péclet number and η is the aspect ratio; that is,

$$\begin{gathered} E_k = \frac{\nu}{2R^2\Omega}, \qquad R_{oT} = \frac{\alpha g\, \Delta T}{\Omega^2 r_0}, \qquad F = \frac{\Omega^2 r_0^2}{gR}, \\ Pe = \frac{\Omega R^2}{\kappa}, \qquad \eta = \frac{R}{r_0}. \end{gathered} \tag{4}$$

The Burger number B_u is given by

$$B_u = \frac{\Delta\rho}{\rho} \frac{gR}{4\Omega^2 L^2} \ .$$

For the Boussinesq fluid we have $\Delta\rho = \rho_0 \alpha \Delta T$; thus in this context the Burger number is

$$B_u = \frac{\alpha g R \, \Delta T}{4\Omega^2 r_0^2} = \frac{\eta R_{oT}}{4}.$$

Table 1 contains values of the relevant parameters, and their respective approximate values in atmospheric and oceanic flows (assuming eddy viscosity in the place of molecular viscosity). In addition, our results assume $F = 1.8 \times 10^{-6}$ and $P_e = 2.1 \times 10^2$.

Parameter	Model	Atmosphere	Ocean
Ekman number E_k	0.017	10^{-4}	10^{-6}
aspect ration η	0.55	0.01	0.01
Burger number B_u	0.10	0.2	O(1)

TABLE 1: Model and geophysical parameter values.

The Navier-Stokes Boussinesq equations, written in spherical coordinates, were solved numerically for the steady-state solution over a wide range of parameters. For details of the numerical methodology; see [**12**]. In particular, for various (fixed) values of the rotation rate Ω and the gap width R, we use pseudo-arclength continuation with the Keller correction condition [**4**] to compute branches of steady solutions for which the differential heating ΔT is made to vary. This corresponds to varying the thermal Rossby number R_{oT} in the nondimensional equations (3); see (4). In this way, effects due to the variation of ΔT may be studied. When $\Delta T = 0$, regardless of the values of the other parameters, there is no convection and the model equations are satisfied by the trivial solution (the equilibrium fluid is stationary in the rotating frame). This provides the starting condition for continuation along the branch. In order to detect bifurcations along the branch of steady-state solutions, we compute the linear stability which is determined by the eigenvalues of the linearization of the equations about the steady-state solution. The eigenvalues are approximated using implicitly restarted Arnoldi iteration following a generalized Cayley transformation; see [**12**], where it is shown that axisymmetric solutions as presented here may be stable even to nonaxisymmetric perturbations, but there is also a possibility of bifurcations to symmetry-breaking solutions.

Typical results of the calculations are shown in Figures 1, 2 and 3, in which the equator lies on the x-axis and the north pole is directed along the y-axis. The angular rotation rate is $\Omega = 0.01$ sec^{-1}, the inner radius is $r_a = 10$ cm, the gap width is $R = 5.5$ cm and the reference temperature is $T_r = 20°$C. When $\Delta T = R_{oT} = 0$, there is no Hadley cell. For very small $R_{oT} > 0$, a single large Hadley cell extends from the equator to the polar region as shown in the first frames of Figure 1. As R_{oT} is increased, a second and third cell (alternating in direction of rotation) enter between this Hadley cell and the pole. The original Hadley cell shrinks toward the equator, while the velocity of the Hadley circulation increases. Note the sensitivity of these cells to small changes of R_{oT} in Figure 1. This behaviour is robust across a wide range of parameter values in the model.

It is of interest that this transition occurs without a bifurcation occurring in the sense that, as ΔT is increased, there is no point at which the solution is neutrally stable; i.e., no eigenvalue crosses the imaginary axis. Other studies, in which flow transitions in systems with a lack of symmetry were investigated, have revealed that such behaviour may be induced by a broken pitchfork bifurcation or a perturbed hysteresis bifurcation [**8, 12, 17, 22**]. Of particular relevance is the fact that [**12**] showed, in a model differing only in the outer boundary condition from the one discussed here, that such a transition resulted from a perturbation of a hysteresis (or cusp) bifurcation. Thus, the observed transition is a fundamentally nonlinear phenomenon. We do not pursue the hysteresis bifurcation in this paper because it is described in detail in [**12**].

The behaviour shown in Figure 1 may be described alternatively as follows: if the temperature gradient ΔT (i.e., R_{oT}) is *decreased* from larger values, then the Hadley cell *expands* poleward while remaining in contact with the equator, and the circulation velocity of the Hadley cell *decreases*. These changes in the Hadley cell occur rapidly with small decreases in ΔT. This is the same type of qualitative behaviour as currently observed for the Hadley cells of Earth. The second and third convection cells that appear in Figure 1, as ΔT increases, resemble the Ferrel and polar cells of the atmospheric circulation [**2, 6**].

In the triptychs of Figures 2 and 3 the three panels show respectively, in a meridional cross section: the meridional stream function (as in Figure 1), the zonal flow relative to the rotating Earth, and the temperature isotherms. The first panel of Figure 3 may be interpreted as showing a strong Hadley cell near the equator, a Ferrel cell at mid-latitudes and a weak polar cell near the north pole, similar to what is observed on Earth. In both Figures 2 and 3, a westerly "jet stream" is apparent in the middle panel at mid-latitudes and high elevation, while surface easterly "trade winds" blow toward the equator at low latitudes. (The

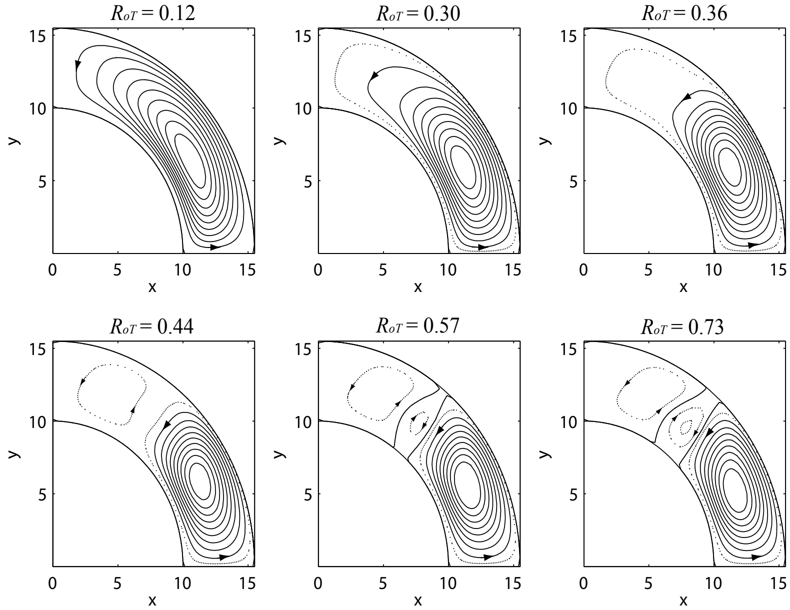

FIGURE 1: Contours of the meridional stream function showing the transition from one Hadley cell to three cells, as the thermal Rossby number R_{oT} (or temperature difference ΔT) is increased. The dashed lines represent contours with one-third the increment of the solid contours. The component of the flow velocity in the meridional plane tends to follow these contours.

+ sign for zonal flow indicates that the azimuthal velocity is in the same direction as the planetary rotation.) Note that the jet stream has moved closer to the equator with the increase in ΔT (i.e., R_{oT}) from Figure 2 to Figures 3; this is a robust feature of the model. The isotherms in the third panel of each figure show that the local temperature gradient remains oriented principally in the north-to-south direction, even as the fluid flow increases. The + sign in these panels indicates temperatures T above the reference temperature T_r. The three panels of Figure 3 compare favorably with the reanalyses of 1979–2001 weather data for the zonal mean meridional stream function, the zonal mean zonal wind and the zonal mean temperature isotherms, respectively, by the European Centre for Medium Range Weather Forecasts (ECMWF), presented in

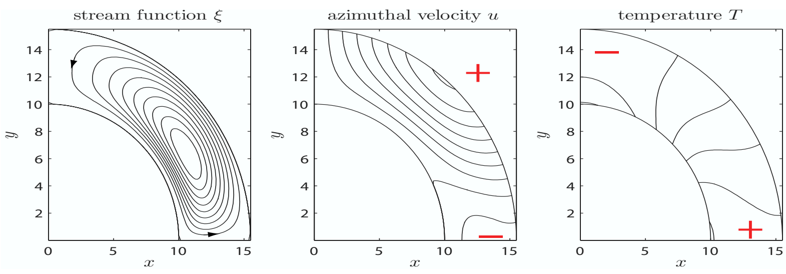

FIGURE 2: Model solution with a single Hadley cell, $R_{oT} = 0.12$ K.

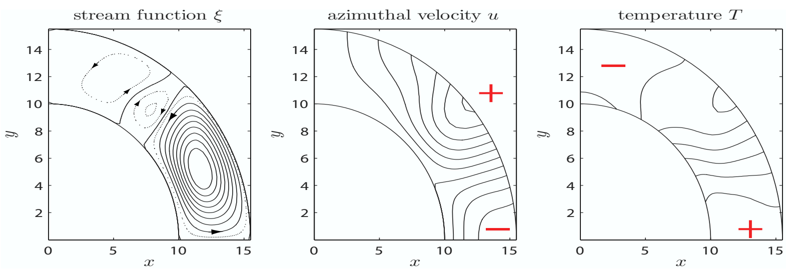

FIGURE 3: Model solution with a three-cell flow, $R_{oT} = 0.73$ K.

their ERA-40 Atlas [**9**].

3 Discussion In the model, both the weakening and the poleward expansion of the Hadley cells that occur with a decrease in ΔT are easily explained. In the general vorticity equation [**6**], the baroclinic vorticity (or solenoidal) term is given by

$$\frac{\partial \vec{\omega}_b}{\partial t} = \frac{1}{\rho^2} \nabla \rho \times \nabla P, \tag{5}$$

where ρ is density, P is pressure, ∇ is the gradient operator and $\vec{\omega}_b$ is the baroclinic vorticity vector. For a Boussinesq fluid, $\rho = \rho_0(1 - \alpha(T - T_r))$, where α is a proportionality constant and T_r is the reference temperature. Thus, (5) becomes

$$\frac{\partial \vec{\omega}_b}{\partial t} = -\left(\frac{\alpha \rho_0}{\rho^2}\right) \nabla T \times \nabla P. \tag{6}$$

The zonal symmetry forces both of the gradients ∇T and ∇P to lie in the meridional plane. For small $\mathbf{u}$, $-\nabla T$ generally points poleward as seen in the third panel of Figure 2 and ∇P is radially inward. Therefore, $-\nabla T \times \nabla P$ points westward out of the meridional plane. By Stokes' Theorem restricted to a meridional plane, the integral of the normal component of vorticity over a closed, simply connected area A is equal to the *circulation* (i.e., the line integral of the tangential component of the velocity vector) around the boundary of A. Thus, the baroclinic vorticity forces a counterclockwise circulation of the Hadley cell as seen in Figures 1 and 2 for small ΔT. The Coriolis force then drives the zonal wind component seen in the middle panel of Figure 2.

An increase in ΔT causes an increase in ∇T (see the third panels of Figures 2 and 3), which increases $\vec{\omega}_b$ from (6) and thus the velocity $\mathbf{u}$ of the Hadley circulation increases. However, if a Hadley cell extended from equator to pole, its zonal circumference would be very large at the equator and would shrink to zero at the pole. Then the poleward flow along the top surface of the Hadley cell would converge to a point at the pole, implying that the flow velocity would approach infinity at the pole. This is impossible in a real fluid. The combination of poleward convergence and increasing baroclinic vorticity forces the poleward circulation to turn downward at finite velocity, well before reaching the pole. Thus the poleward extent of the Hadley cell *decreases* and the velocity $\mathbf{u}$ of the Hadley circulation *increases*, as the baroclinic vorticity $\vec{\omega}_b$ increases with ΔT. In the model, this dependence on ΔT is very strong. Furthermore, as seen in Figures 2 and 3, the jet stream in the model moves toward the equator with increasing ΔT, in accord with recent observations in the Earth's atmosphere [**1, 7**].

The circulation of the atmosphere of the Earth is affected by many additional factors not present in this simple model. The model omits all the direct effects of greenhouse gases, water vapor and latent heat, oceans and ocean heat transport, continents and mountains, vegetation, albedo, radiant heating, turbulent mixing and more. For example, the trade winds pick up large quantities of moisture as they approach the equator. The latent heat released as rain forms in the ITCZ is an important source of additional energy for the Hadley cells of Earth. Zonal asymmetries cause significant perturbations to the Hadley cell patterns on Earth, but the zonally- and temporally-averaged atmospheric flow patterns of the ERA-40 Atlas are remarkably similar to those of the zonally symmetric time-independent model in Figure 3. For the atmosphere, the density ρ is a function of pressure P as well as of temperature T; however, the T dependence is of fundamental importance in determining the circulation of Hadley cells. Baroclinic eddy fluxes are a significant mechanism for energy and mass transport in the Earth's atmosphere outside of the tropics and influence the Hadley circulation [**27**]. Turbulent diffusion is simulated by thermal diffusion in the model. In [**14**] it has been suggested that poleward shifts in midlatitude circulation are predominantly driven by a rise in height of the tropopause, a factor ignored in this model. The success of the model in providing a simple explanation for both the weakening and the expansion of the Hadley cells, and the movement of the jet streams, suggests that all of these differences are of secondary importance, and the fundamental convection mechanism driven by baroclinic vorticity and convergence instability that determines the circulation pattern in the model may also be the primary mechanism for the Hadley cell expansion currently underway on Earth.

Currently there is much interest in the observed changes in the Hadley cells and their causes. Initial reports suggested that there has been a "strengthening" of the Hadley circulation in recent years, which would contradict the behaviour of this convection model. Mitas and Clement found some evidence for a strengthening of the Hadley circulation, but also significant discrepancies between GCM simulations and data reanalyses, with a majority of 94 GCM simulations showing no increase of Hadley cell strength [**19, 20**]. Today, the evidence of poleward expansion of the Hadley cells in [**3, 7, 16**] is widely accepted. Lu *et al.* [**16**] confirmed not only the poleward expansion but also a consistent weakening (i.e., slowing) of the Hadley circulation across the IPCC AR4 simulations. Hu and Fu [**7**] suggested qualitatively that weaker meridional temperature gradients could cause Hadley cell expansion and that current GCMs may not have the capability to well simulate changes in Hadley circulation.

Since the model of this paper is based on fundamental physics and geometry, and does not rely on any specific weather data, it applies generally to the atmospheres of other planets. Earth's sister planet Venus has similar mass, radius and solar radiation, to those of the Earth. However, the atmosphere of Venus is mostly carbon dioxide (96.5%), with clouds of sulfuric acid and very little water. The surface temperature and pressure are much higher than on Earth, 467°C and 93 bar, respectively. The rate of rotation is very slow, with a sidereal period of 243 Earth days. Venus is the ultimate greenhouse gas planet. Observations by the Venus Express and other spacecraft have brought detailed information on conditions of the surface and the lower atmosphere of Venus [**26**]. The troposphere on Venus is 65 km deep. Under this thick blanket, the meridional surface temperature gradient is smaller than on Earth. The tropospheric circulation of Venus is dominated by two large Hadley cells of velocity less than 2 ms^{-1} at the surface, and extending from the equator to about $\pm 60°$ latitude. These Venusian "greenhouse" Hadley cells are much larger and slower than those on Earth, in agreement with the behaviour of our model.

4 Conclusions and future work It has been noted that the climate of the Earth is determined by many factors in addition to the convection mechanism studied here. The fact that this model successfully accounts for both the weakening and the expansion of the Hadley cells, as well as the poleward movement of the jet stream, employing only the fundamental physics of convection and diffusion in a rotating spherical geometry, suggests that those other factors may be of only secondary importance. The current differential warming of the polar regions seems to be adequately explained by positive feedback effects. The resulting decrease in temperature gradient in turn decreases the convective forcing, which in the model causes changes very similar to those observed in the Hadley cells and jet streams of Earth. The model is far from providing a full explanation for these observed climate changes. It merely indicates that perhaps greater attention should be paid to the role of convection in these changes than has been the case.

Further work is required to test the implications of this simple model, which is but a first step toward a model with a more Earth-like atmosphere on a planetary scale. Replacing the Boussinesq fluid with a compressible fluid (e.g., ideal gas) will introduce vertical stratification of the model atmosphere as is the case on Earth. This would lead to a smaller gap width R in the model that corresponds better with the very thin shell that is the Earth's atmosphere. A two-parameter study of the behaviour of the Hadley cells, as both the global mean temperature T_M and the pole-to-equator difference ΔT are varied independently, is planned. Non-axisymmetric solutions have not been described here but are known to exist, and would allow the model to reproduce, for example, the latitudinal meandering of the jet stream. Similarly, solutions that break the north-south symmetry remain to be explored. The positive feedback effect of the heat that is transported by the Hadley cells themselves has not been included in the model. These and other extensions of the model will be reported elsewhere. The extreme sensitivity to small changes in the temperature gradient exhibited by the Hadley cells in this simple model strongly suggests that the dependence on ΔT

will persist in these enhanced models.

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