

NONLINEAR WAVES IN MAGNETIC FIELD

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ABSTRACT. Employing the method of multiple time scales, we derive here the transport equations for the primary amplitudes of resonantly interacting high frequency planar waves propagating into perfectly conducting fluids permeated by a transverse magnetic field. The evolutionary behavior of nonresonant wave modes culminating into shock waves is also studied.

1 Introduction When a piston begins to move into an inviscid gas that is initially at rest, pressure waves propagate in the gas ahead of the piston and the gas is set in motion by the waves. The equations are nonlinear, and the wave velocity depends on the solution values (higher where pressure is higher, for example). This causes the waveform to distort, leading to steepening of the parts of the wave carrying compression and eventual shock formation.

The effects of nonlinearity on wave propagation in gases have been much studied in the past. The work of Kumar *et al.* [13], Hunter and Ali [12], Whitham [19], Moodie *et al.* [16], He and Moodie [6]–[9], Sharma and Arora [17], Arora and Sharma [2], and Arora [1] is worth mentioning in this context. We use Asymptotic method for the analysis of weakly nonlinear hyperbolic waves. Choquet-Bruhat [4] proposed a method to discuss shockless solutions of hyperbolic systems which depend upon a single phase function. Germain [5] has given the general discussion of single phase progressive waves. Hunter and Keller [10] established a general nonresonant multi-wave mode theory which has led to several interesting generalizations by Majda and Rosales [14] and Hunter *et al.* [11] to include resonantly interacting multi-wave mode features.

We use the resonantly interacting multi-wave theory to examine small amplitude high frequency asymptotic waves for one-dimensional unsteady planar motion of a general inviscid ideal gas permeated by a transverse magnetic field, where at the leading order many waves co-exist and interact with one another resonantly, and obtain evolution

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equations which describe the resonant wave interactions inherent in the system.

2 Basic equations Assuming the electrical conductivity to be infinite and the direction of the magnetic field orthogonal to the trajectories of the fluid particles, the basic equations for a one-dimensional planar motion of an ideal gas are (Whitham [19])

$$(1) \quad \begin{aligned} \frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x} &= 0, \\ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \left(\frac{\partial p}{\partial x} + \frac{\partial h}{\partial x} \right) &= 0, \\ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + \rho a^2 \frac{\partial u}{\partial x} &= 0, \\ \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + 2h \frac{\partial u}{\partial x} &= 0, \end{aligned}$$

where u is the fluid velocity, p is the pressure, ρ is the density, $h = \mu H^2/2$ is the magnetic pressure with H as the transverse magnetic field strength and μ as the magnetic permeability, $a = (\gamma p/\rho)^{1/2}$ is the speed of sound with γ as the adiabatic exponent, t is the time, and x is the spatial coordinate. Also, all the variables in system (1) are dimensionless.

The equations (1) may be written in matrix form as

$$(2) \quad \frac{\partial U}{\partial t} + A \frac{\partial U}{\partial x} = 0$$

where U is the column vector defined by

$$(3) \quad U = (\rho, u, p, h)^{tr}.$$

A is the 4 x 4 matrix having the components A^{ij} , the nonzero ones are as follows:

$$(4) \quad \begin{aligned} A^{11} = A^{22} = A^{33} = A^{44} &= u, \\ A^{12} = \rho, \quad A^{23} = A^{24} = \frac{1}{\rho}, \quad A^{32} = \rho a^2, \quad A^{42} &= 2h \end{aligned}$$

System (2) being strictly hyperbolic admits four families of characteristics, among them two represent waves propagating in $\pm x$ directions

with the speed $u \pm c$, where $c = (a^2 + b^2)^{1/2}$ represents the magnetoacoustic speed with $b = (2h/\rho)^{1/2}$ as the Alfvén speed. The remaining two families form a set of double characteristics representing entropy waves or particle paths propagating with velocity u . We consider waves propagating into a constant background state $U_0 = (\rho_0, 0, p_0, h_0)^{tr}$. The characteristic speeds at $U = U_0$ are given by $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = c_0$ and $\lambda_4 = -c_0$. The subscript 0 refers to evaluation at $U = U_0$, and is synonymous with a state of equilibrium.

3 Interaction of high frequency waves We denote the left and right eigenvectors of A_0 associated with the eigenvalue λ_i by $L^{(i)}$ and $R^{(i)}$. These eigenvectors satisfy the normalization condition $L^{(i)} R^{(j)} = \delta_{ij}$, $1 \leq i, j \leq 4$, where δ_{ij} represents the Kronecker's delta. These eigenvectors are given as

$$\begin{aligned}
 L^{(1)} &= \left(1, 0, -\frac{1}{c_0^2}, -\frac{1}{c_0^2}\right), & R^{(1)} &= (1, 0, a_0^2, -a_0^2)^{tr}, \\
 L^{(2)} &= (-a_0^2, 0, 1, 0), & R^{(2)} &= (0, 0, 1, -1)^{tr}, \\
 (5) \quad L^{(3)} &= \left(0, \frac{\rho_0}{2c_0}, \frac{1}{2c_0^2}, \frac{1}{2c_0^2}\right), & R^{(3)} &= \left(1, \frac{c_0}{\rho_0}, a_0^2, b_0^2\right)^{tr}. \\
 L^{(4)} &= \left(0, -\frac{\rho_0}{2c_0}, \frac{1}{2c_0^2}, \frac{1}{2c_0^2}\right), & R^{(4)} &= \left(1, -\frac{c_0}{\rho_0}, a_0^2, b_0^2\right)^{tr}.
 \end{aligned}$$

We look for asymptotic solution for (2) as $\epsilon \rightarrow 0$ of the form

$$(6) \quad U \sim U_0 + \epsilon U_1(x, t, \tilde{\theta}) + \epsilon^2 U_2(x, t, \tilde{\theta}) + O(\epsilon^3),$$

where U_1 is a smooth bounded function of its arguments and U_2 is bounded in (x, t) in a certain bounded region of interest having at most sub-linear growth in θ as $\theta \rightarrow \pm\infty$. Here $\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4)$ represents the "fast variables" characterized by the functions ϕ_i as $\theta_i = \phi_i/\epsilon$, where ϕ_i , $1 \leq i \leq 4$, is the phase of the i -th wave associated with the characteristic speed λ_i . Now we use (6) in (2), expand A in Taylor's series in powers of ϵ about $U = U_0$, replace the partial derivatives $\partial/\partial X$ (X being either x or t) by $\frac{\partial}{\partial X} + \epsilon^{-1} \sum_{i=1}^4 \frac{\partial \phi_i}{\partial X} \frac{\partial}{\partial \theta_i}$, and equate to zero the

coefficients of ϵ^0 and ϵ^1 in the resulting expansions, to obtain

$$(7) \quad O(\epsilon^0) : \sum_{i=1}^4 \left(I \frac{\partial \phi_i}{\partial t} + A_0 \frac{\partial \phi_i}{\partial x} \right) \frac{\partial U_1}{\partial \theta_i} = 0$$

$$(8) \quad O(\epsilon^1) : \sum_{i=1}^4 \left(I \frac{\partial \phi_i}{\partial t} + A_0 \frac{\partial \phi_i}{\partial x} \right) \frac{\partial U_2}{\partial \theta_i} = -\frac{\partial U_1}{\partial t} - A_0 \frac{\partial U_1}{\partial x} \\ - \sum_{i=1}^4 \frac{\partial \phi_i}{\partial x} (U_1 \cdot \nabla A)_0 \frac{\partial U_1}{\partial \theta_i}$$

where I is the 4×4 unit matrix and ∇ is the gradient operator with respect to the dependent variable U . Now since the phase functions ϕ_i , $1 \leq i \leq 4$, satisfy the eikonal equation

$$(9) \quad \text{Det} \left(I \frac{\partial \phi_i}{\partial t} + A_0 \frac{\partial \phi_i}{\partial x} \right) = 0,$$

we choose the simplest phase function of this equation, namely

$$(10) \quad \phi_i(x, t) = x - \lambda_i t, \quad 1 \leq i \leq 4.$$

It follows from (5) that for each phase ϕ_i , $\partial U_1 / \partial \theta_i$ is parallel to the right eigen vector $R^{(i)}$ of A_0 and thus

$$(11) \quad U_1 = \sum_{i=1}^4 \sigma_i(x, t, \theta_i) R^{(i)},$$

where $\sigma_i = (L^{(i)} \cdot U_1)$ is a scalar function called the wave amplitude, that depends only on the i -th fast variable θ_i . We assume that $\sigma_i(x, t, \theta_i)$ has zero mean value with respect to the fast variable θ_i , that is,

$$(12) \quad \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \sigma_i(x, t, \theta_i) d\theta_i = 0.$$

We then use (11) in (8) and solve for U_2 . To begin with we write

$$U_2 = \sum_{j=1}^4 m_j R^{(j)},$$

substitute this value in (8), and premultiply the resulting equation by $L^{(i)}$ to obtain the system of decoupled inhomogeneous first order partial differential equations:

$$(13) \quad \sum_{j=1}^4 (\lambda_i - \lambda_j) \frac{\partial m_i}{\partial \theta_j} = -\frac{\partial \sigma_i}{\partial t} - \lambda_i \frac{\partial \sigma_i}{\partial x} - \sum_{j=1}^4 L^{(i)}(U_1 \cdot \nabla A)_0 \frac{\partial U_1}{\partial \theta_j},$$

$$1 \leq i \leq 4.$$

The characteristic ODEs for the i th equation in (3) are given by

$$(14) \quad \dot{\theta}_j = \lambda_i - \lambda_j \quad \text{for } j \neq i, \quad \dot{\theta}_i = 0, \quad \dot{m}_i = H_i,$$

where

$$H_i(x, t, \theta_1, \theta_2, \theta_3, \theta_4) = -\frac{\partial \sigma_i}{\partial t} - \lambda_i \frac{\partial \sigma_i}{\partial x} - \sum_{j=1}^4 L^{(i)}(U_1 \cdot \nabla A)_0 \frac{\partial U_1}{\partial \theta_j}.$$

We asymptotically average (3) along the characteristics and appeal to the sub-linearity of U_2 in θ , which ensures that the expression (6) does not contain secular terms. The constancy of θ_i along the characteristics and the vanishing asymptotic mean value of \dot{m}_i along the characteristics implies that the wave amplitudes σ_i , $1 \leq i \leq 4$, satisfy the following system of coupled integro-differential equations

$$(15) \quad \frac{\partial \sigma_i}{\partial t} + \lambda_i \frac{\partial \sigma_i}{\partial x} + \Gamma_{ii}^i \sigma_i \frac{\partial \sigma_i}{\partial \theta_i} + \sum_{i \neq j \neq k} \Gamma_{jk}^i \lim_{T \rightarrow \infty} \frac{1}{2T}$$

$$\times \int_{-T}^T \sigma_j(\theta_i + (\lambda_i - \lambda_j)s) \sigma_k(\theta_i + (\lambda_i - \lambda_k)s) ds = 0,$$

where $\sigma_k = \partial \sigma_k / \partial \theta_k$, and the coefficient Γ_{jk}^i is given by

$$(16) \quad \Gamma_{jk}^i = L^{(i)}(R^{(j)} \cdot \nabla A)_0 R^{(k)}.$$

The interaction coefficients Γ_{jk}^i , which are asymmetric in j and k , denote the strength of coupling between the j -th and k -th wave modes ($j \neq k$) that can generate an i -th wave ($i \neq j \neq k$). The coefficients Γ_{ii}^i which refer to the non-linear self-interaction, are nonzero for genuinely nonlinear waves and zero for linearly degenerate waves. It is also interesting

to note that if all the coupling coefficients Γ_{jk}^i ($i \neq j \neq k$) are zero or the integral in (15) vanishes, the waves do not resonate and (15) reduces to a system of uncoupled Burgers' equations. The coefficients Γ_{ii}^i and Γ_{jk}^i , given by (15), provide a picture of the non-linear interaction process present in the system under consideration, and can be easily determined in the following form; the non-zero ones being:

$$\begin{aligned}
\Gamma_{23}^1 &= -\Gamma_{24}^1 = \frac{2-\gamma}{\rho_0 c_0}, \\
\Gamma_{34}^1 &= -\Gamma_{43}^1 = \frac{c_0^2 + a_0^2(\gamma-2)}{\rho_0 c_0}, \\
(17) \quad \Gamma_{13}^2 &= \Gamma_{43}^2 = -\Gamma_{14}^2 = -\Gamma_{34}^2 = (\gamma-1)a_0^2 \frac{c_0}{\rho_0}, \\
\Gamma_{14}^3 &= -\Gamma_{13}^4 = -\frac{c_0^2 + a_0^2(\gamma-2)}{2\rho_0 c_0} = \beta_1, \\
\Gamma_{24}^3 &= -\Gamma_{23}^4 = \frac{(2-\gamma)}{2\rho_0 c_0} = \beta_2.
\end{aligned}$$

The resonance equations (15) can now be written as

$$\begin{aligned}
(18) \quad & \frac{\partial \sigma_1}{\partial t} = 0, \quad \frac{\partial \sigma_2}{\partial t} = 0, \\
& \frac{\partial \sigma_3}{\partial t} + c_0 \frac{\partial \sigma_3}{\partial x} + \Gamma \sigma_3 \frac{\partial \sigma_3}{\partial \theta_3} \\
& - \lim_{P \rightarrow \infty} \frac{1}{2P} \int_{-P}^P K\left(x, t, \frac{\theta_3 + \phi}{2}\right) \sigma_4(x, t, \phi) d\phi = 0, \\
& \frac{\partial \sigma_4}{\partial t} - c_0 \frac{\partial \sigma_4}{\partial x} - \Gamma \sigma_4 \frac{\partial \sigma_4}{\partial \theta_4} \\
& + \lim_{P \rightarrow \infty} \frac{1}{2P} \int_{-P}^P K\left(x, t, \frac{\theta_4 + \phi}{2}\right) \sigma_3(x, t, \phi) d\phi = 0,
\end{aligned}$$

where $\Gamma = \Gamma_{33}^3 = -\Gamma_{44}^4 = [3c_0^2 - (2-\gamma)a_0^2]/(2c_0\rho_0)$, and the kernel K is defined as

$$(19) \quad K\left(x, t, \frac{\theta + \phi}{2}\right) = \frac{\beta_1}{2} \frac{\partial \sigma_1}{\partial \theta_1}\left(x, t, \frac{\theta + \phi}{2}\right) + \frac{\beta_2}{2} \frac{\partial \sigma_2}{\partial \theta_2}\left(x, t, \frac{\theta + \phi}{2}\right),$$

with β_1 and β_2 being the same as in (17).

Let the initial value of σ_j be $\sigma_j|_{t=0} = \sigma_j^0(x, \theta_j)$. Hence (18)_{1,2} gives $\sigma_1(x, t, \theta_1) = \sigma_1^0(x, \theta_1)$ and $\sigma_2(x, t, \theta_2) = \sigma_2^0(x, \theta_2)$, and subsequently the system (18) reduces to a pair of equations for the wave fields σ_3 and σ_4 coupled through the linear integral operator involving the kernel

$$(20) \quad K(x, t, \theta) = \frac{\beta_1}{2} \frac{\partial \sigma_1^0}{\partial \theta_1}(x, \theta) + \frac{\beta_2}{2} \frac{\partial \sigma_2^0}{\partial \theta_2}(x, \theta).$$

If the initial data $\sigma_j^0(x, \theta)$ are 2π -periodic function of the phase variable θ , then the pair of resonant asymptotic equations in system (18) become

$$(21) \quad \begin{aligned} \frac{\partial \sigma_3}{\partial t} + c_0 \frac{\partial \sigma_3}{\partial x} + \Gamma \sigma_3 \frac{\partial \sigma_3}{\partial \theta_3} \\ - \frac{1}{2\pi} \int_{-\pi}^{\pi} K \left(x, t, \frac{\theta_3 + \phi}{2} \right) \sigma_4(x, t, \phi) d\phi = 0, \\ \frac{\partial \sigma_4}{\partial t} - c_0 \frac{\partial \sigma_4}{\partial x} - \Gamma \sigma_4 \frac{\partial \sigma_4}{\partial \theta_4} \\ + \frac{1}{2\pi} \int_{-\pi}^{\pi} K \left(x, t, \frac{\theta_4 + \phi}{2} \right) \sigma_3(x, t, \phi) d\phi = 0, \end{aligned}$$

where K is given by (20).

The system of equations obtained in equation (21) is (mathematically) the same system obtained in [14], for the compressible Euler equations of Gas Dynamics. The thing that changes is the physical meaning of the various parameters (coupling kernel K in the integral term, wave velocities, etc.) because of the addition of the magnetic field. The behavior of this system (including shocks even in the resonant case) was studied extensively in [15].

More details (for the special case when the wave interactions occur within a closed pipe) can be found in [18]. Shocks in the resonant case within a bounded environment seem to be very important, as they lead the solutions to asymptote to very nontrivial nonlinear standing wave like states.

4 Nonlinear geometrical acoustics solution The asymptotic solution (6) of hyperbolic system (2) satisfying small amplitude oscillating initial data

$$(22) \quad U(x, 0) = U_0 + \epsilon U_1^0(x, x/\epsilon) + O(\epsilon^2),$$

is non-resonant if $U_1^0(x, x/\epsilon)$ are smooth functions with a compact support [14]. The issue of how to deal with shocks when resonances occur is studied in Cehelsky and Rosales [3].

The characteristic equations are

$$(23) \quad \frac{d\theta_j}{dx} = \frac{\Gamma\sigma_j}{c_0}, \quad \frac{dt}{dx} = \frac{e_j}{c_0}, \quad j = 3, 4$$

where

$$e_j = \begin{cases} +1 & \text{if } j = 3, \\ -1 & \text{if } j = 4. \end{cases}$$

In terms of the characteristic equations, the decoupled equations (21)₂ and (21)₃ can be written as

$$(24) \quad \frac{d\sigma_j}{dt} = 0, \quad j = 3, 4$$

which yields on integration

$$(25) \quad \sigma_j = \sigma_j^0(s_j, \xi_j), \quad j = 3, 4$$

along the rays $s_j = x - e_j c_0 t = \text{constant}$, where the function σ_j^0 ($j = 3, 4$) is obtained from the initial condition (22), and the fast variable ξ_j parametrizes the set of characteristic curves (23)₁.

Thus, we obtain from (23)

$$(26) \quad \xi_j = \theta_j - e_j \Gamma \sigma_j^0(s_j, \xi_j) t, \quad j = 3, 4$$

The solution of (2), satisfying (22), where $U_1^0(x, x/t)$ has compact support, is obtained as

$$(27) \quad \begin{aligned} \rho(x, t) &= \rho_0 + \epsilon \sigma_1^0(x, x/\epsilon) + \epsilon (\sigma_3^0(s_3, \xi_3) + \sigma_4^0(s_4, \xi_4)) + O(\epsilon^2), \\ u(x, t) &= \epsilon \frac{c_0}{\rho_0} (\sigma_3^0(s_3, \xi_3) - \sigma_4^0(s_4, \xi_4)) + O(\epsilon^2), \\ p(x, t) &= p_0 + \epsilon a_0^2 \sigma_1^0(x, x/\epsilon) + \epsilon \sigma_2^0(x, x/\epsilon) \\ &\quad + \epsilon a_0^2 (\sigma_3^0(s_3, \xi_3) + \sigma_4^0(s_4, \xi_4)) + O(\epsilon^2), \\ h(x, t) &= h_0 - \epsilon a_0^2 \sigma_1^0(x, x/\epsilon) - \epsilon \sigma_2^0(x, x/\epsilon) \\ &\quad + \epsilon b_0^2 (\sigma_3^0(s_3, \xi_3) + \sigma_4^0(s_4, \xi_4)) + O(\epsilon^2), \end{aligned}$$

where the fast variables ξ_j ($1 \leq j \leq 4$) are given in (26), and the initial values for σ_i ($1 \leq i \leq 4$) are obtained from the solution (27) specified at $t = 0$ as

$$\begin{aligned}
 \sigma_1^0\left(x, \frac{x}{\epsilon}\right) &= \rho_1^0\left(x, \frac{x}{\epsilon}\right) - \left(\frac{1}{c_0^2}\right)\left(p_1^0\left(x, \frac{x}{\epsilon}\right) + h_1^0\left(x, \frac{x}{\epsilon}\right)\right), \\
 \sigma_2^0\left(x, \frac{x}{\epsilon}\right) &= -a_0^2 \rho_1^0\left(x, \frac{x}{\epsilon}\right) + p_1^0\left(x, \frac{x}{\epsilon}\right), \\
 \sigma_3^0(x, \xi_3) &= \left(\frac{\rho_0}{2c_0}\right)u_1^0(x, \xi_3) + \left(\frac{1}{2c_0^2}\right)(p_1^0(x, \xi_3) + h_1^0(x, \xi_3)), \\
 \sigma_4^0(x, \xi_4) &= -\left(\frac{\rho_0}{2c_0}\right)u_1^0(x, \xi_4) + \left(\frac{1}{2c_0^2}\right)(p_1^0(x, \xi_4) + h_1^0(x, \xi_4)).
 \end{aligned}
 \tag{28}$$

This is the complete solution of (2) and (22); any multivalued overlap in this solution is resolved by introducing shocks into the solution.

5 Shock waves Following [10] it can be shown that the shock location θ_j^s satisfies the relation

$$\frac{d\theta_j^s}{dt} = \frac{1}{2}\Lambda_{jj}^j\left(\sigma_j^{(-)} + \sigma_j^{(+)}\right), \quad j = 3, 4
 \tag{29}$$

which is the shock speed in θ_j - t plane. Here $\sigma_j^{(-)}$ and $\sigma_j^{(+)}$, respectively, are the values of σ_j just ahead and behind the shock. We have $\sigma_j^{(-)} = 0$ for the undisturbed region ahead of the shock. Now we use (25) and drop the superscripts on θ_j^s and σ_j^+ to obtain

$$\frac{d\theta_j}{dt} = \frac{\Gamma}{2}e_j\sigma_j^0(s_j, \xi_j).
 \tag{30}$$

Now using (30) and (26) we obtain the following equation which determines the shock path parametrically,

$$\theta_j = \xi_j - \frac{2}{\sigma_j^0} \int_0^{\xi_j} \sigma_j^0(\tilde{t}) d\tilde{t}.
 \tag{31}$$

If $\sigma_j^0 \neq 0$, then the shock forms right at the origin.

6 Conclusion The method of multiple scales is used to obtain small amplitude high frequency asymptotic solution to the basic equations governing one dimensional unsteady planar flow in an ideal gas, where it is assumed that the electrical conductivity is infinite, and the direction of the magnetic field is orthogonal to the trajectories of the fluid particles. Weakly nonlinear geometrical acoustics theory is used to analyze the resonant waves interaction. We derived the transport equations for the wave amplitudes along the rays of the governing system; these transport equations constitute a system of inviscid Burger's equations with quadratic nonlinearity coupled through a linear integral operators with a known kernel. The coefficients appearing in the transport equations provide a measure of coupling between the various modes and set a qualitative picture of the interaction process involved therein. It is observed that the wave fields associated with the particle paths do not interact with each other; however they do interact with an acoustic wave field to produce resonant contribution towards the other acoustic field. The acoustic wave fields may or may not interact, but in either case their net contribution, which is directed towards the entropy field, is always zero. In our analysis the governing system of Euler equations reduces to a pair of resonant asymptotic equations for the acoustic wave fields. For a nonresonant multi wave mode case, proposed by Hunter and Keller [10], the reduced system of transport equations gets decoupled with vanishing integral average terms, and the occurrences of shocks in the acoustic wave fields are analyzed.

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