SIMULATION METHODOLOGY OF MICRO-ELECTRO-MECHANICAL SYSTEMS. PART 1: FULL 3D-MODELS

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ABSTRACT. The paper is dedicated to a state-of-the-art review of the numerical methods used in the simulation of Micro-Electro-Mechanical Systems (MEMS). We treat all the basic approaches that are used for the realization of modern methods of MEMS simulation. We also classify the numerical models in current use. We describe the basic methods used for full three-dimensional simulation of MEMS, and present examples of numerical models of MEMS. We present some prospects for development of systems components suggested by the simulation methods.

1 Introduction The origin of the field of Micro-Electro-Mechanical Systems can be traced to the beginning of the 1980’s [38], when the first papers appeared, opening a path to the study of nano-electronic devices. At present it is possible to find examples of MEMS application in practically every area of instrument making: from the automobile industry [57] and telecommunications industry [7] to microbiology and the development of medical equipment [22, 23, 55]. During the last two decades the study of Micro-Electro-Mechanical Systems has evolved from a field of only academic researches into an integral part of micro- and nano-technological bases for modern instrument making [13, 14, 22, 45]. MEMS-based devices have now numerous applications, such as microsensors, microactuators, microaccelerometers, microphones, cellular phones and microelectromechanical filters. MEMS elements that have appeared to date include rotary motors, linear motors, resonators, springs, gears, grippers, diaphragms and arrays of mirrors for display technology. Now the techniques for creating Micro-Electro-Mechanical Systems are capable of producing composite systems integrally, including micro-mechanical parts, analog circuits and digital logic [11]. The manual development of these types of device is extremely dif-

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icult owing to the interdisciplinary nature of a developed system and the great number of intra-system interactions required of the model. MEMS is now an essential interdisciplinary field of electronics, bringing together studies in mechanical engineering, electrical engineering, electronics, fluid mechanics, optics, chemistry and chemical engineering. The presence of interdisciplinary physical processes in a system under design generally requires the use of hybrid systems simulation tools. The development and use of these tools is quite difficult. The rapid increase in the complexity of systems being designed, and the cutting edge level of this complexity, leads today to the necessity of creating general-system design methods that integrate development tools from each of the relevant fields of physics and engineering [28, 29, 31, 40, 42, 46]. A basic approach, one that is widely used at present for MEMS design, is the hierarchically structured design [31, 42], [12, 27, 30, 39, 41, 46]. Its flowchart is presented in Figure 1.

According to this methodology, the design of the device begins with the stage of design topology. The behavioral simulation of MEMS must be done after this first stage. The multicomponent physical system is split into subsystems and components. MEMS simulation runs in two steps, in the first of which the behavioral simulation of components is modeled. The second step is associated with simulation of MEMS as a whole. After the system simulation stage, the MEMS description is created, and finally the extraction of parasitic capacitances and final verification of the system must be carried out. These stages of MEMS design correspond to the set of tools used in the automatic design of devices. The set of tools for geometric design is one of most important Computer Aided Design (CAD) components [33, 37, 56]. It is necessary to take the production technology into consideration during this stage of MEMS design. This leads in turn to the creation of tools for technological process simulation inside of MEMS CAD programs [18, 19, 24]. Use of these tools, for example with the CAD program CoventorWare (Coventor Inc.), makes possible not only the calculation of device geometric parameters on the basis of technology process characteristics and description of designed system; it also makes it possible to visualize the designed MEMS that will be produced by some definite technology. The paradigm of structured design comes into the MEMS design field from the area of Very Large Scale Integration (VLSI) Systems. Computer Aided Design for VLSI spans many levels of abstraction from materials, devices, circuits, logic, register to the level of the system. At each of these levels a design can be viewed in physical, structural (schematic) or behavioral form. One of the main tasks in development of structured
MEMS design tools is the formation of standard data representations and standard cell libraries.

The list of modern subfields of activity from the area of structured MEMS design includes:

- standard MEMS data representations and interchange formats
- standard MEMS cell libraries supporting behavioral, schematic and physical views at all levels of abstraction. These libraries must include materials database, layout cells, schematic element library, system macro-model library
- standard MEMS process-module libraries and standard process flows
- process simulation and visualization
process synthesis and technology file extraction
- Three-dimensional (3D) rendering and animation
- 3D generation from layout and technology files
- layout of arbitrarily shaped objects with design rule checking
- layout synthesis and verification
- fast modeling and verification tools: coupled multi-domain numerical analysis tools
- parasitic extraction to schematic and behavioral views
- macro-model parameter extraction from physical and schematic views
- multi-domain schematic capture: schematic view of connectivity between mechanical, electromechanical, thermal and circuit lumped-parameter elements
- mixed-signal multi-level multi-domain simulation.

A vital problem for modern CAD systems used for MEMS design today is the poor level of development of general-system simulation methods, the basic features of which are: the low level of general-system verification methods, the limitations placed on the amount of complication allowed in developed devices and the significant increase in expenditures for design involving increases of complexity in MEMS. Methods for the development of mathematical modeling for technical systems, including various physical components, is now the main problem facing CAD in this field of microelectronics. In particular, one of central problems of simulation methods development is the creation of macromodels for
possible MEMS components which precisely describe the dynamics of device functioning.

2 The basic approaches used for development of MEMS simulation methods
There are presently two main directions for the development philosophy of MEMS simulation. The first is the systems approach, in which the architecture of the system is built by a principle of unification, with the integrated use of specialized simulators. Each of these simulators is associated with a definite branch of physics or a definite level of MEMS description for the system being considered. According to the second approach, the heterogeneous physical components of MEMS are considered within the framework of a unified simulation medium on the basis of methods designed for their homogeneous description. Both of these directions are being actively developed at present.

The multicomponent physical system as the core of a MEMS mathematical model can be described by a set of differential equations. For the study of processes which occur in such systems, the methods of full three-dimensional simulation are applied. Three-dimensional models are used both for the in-depth study of the dynamics of a system, and for problem solving directly tied to the development of reduced-order models. The application of 3D-models in numerical analysis is usually associated with considerable expenditure of computer time. The demands for resources indispensable for the numerical analysis of a design can be essentially reduced through the application of reduced-order models (macromodels). Development of methods for full three-dimensional MEMS simulation together with the construction of macromodels constitute two basic branches of modern tools development for the numerical modeling of Micro-Electro-Mechanical Systems. It is necessary to note that the development of macromodeling methods and the development of 3D-models do advance by independent paths. This is associated with the fact that three-dimensional models in some cases are required at level of making macromodels [15]. Secondly, an ideological link between our two approaches also exists at the level of using the same conceptual approaches, as it was, for example, with the use of the method of Multiple-Energy Domain Representation. This approach was presented in [15] as a basis of a method of MEMS macromodel development. Later it was used in the well-known set of programs SUGAR for the development of three-dimensional models [10].
3 Physical and mathematical models of micro-electro-mechanical systems

It is possible to sub-divide into two groups the collection of mathematical interpretations of physical analogs used within the framework of MEMS simulation methods. The first type of mathematical model has in the literature the title of non-lumped parameters models. Within the framework of this type of description, the modeled system is considered as a continuum. The physical properties of a homogeneous system are described by continuous functions. The presence in a simulated system of several different technological materials (in the elementary case) is accounted for through discontinuities of functions at the applicable boundaries. By virtue of its specificity, this approach is used extensively for the development of full three-dimensional models of Micro-Electro-Mechanical Systems. The second type of mathematical model has the title of lumped parameters models. Within the framework of this description the modeled system is considered as combination of discrete elements. Definite physical characteristics are assigned to each of these elements. In particular, each of the systems components can be considered as an absolutely rigid body, or as a deformable body, depending on the properties of the modeled system and demands to be met by the mathematical model. This approach is successfully used both at the development stage for three-dimensional models, and in problems of macromodelling. Use of these two approaches for development of MEMS mathematical models is completely determined by the relevant physics on which is based the functioning principles of the particular device. Accounting for electro-elastostatic or electro-elastodynamic phenomena in the model gives the advantage of using models with lumped parameters. The necessity of analysis of phenomena from the field of air- or hydro-dynamics or adjoining these areas to other areas of physics results in the necessity of using non-lumped parameter models. The convenience and possibility for use of each type of mathematical model is dictated by demands for the realization of an effective mathematical method and the correct formulation of the physical problem.

4 Methods of development of full three-dimensional MEMS models

For the development of full three-dimensional simulations for Micro-Electro-Mechanical Systems both lumped- and non-lumped parameters models can be used. The type of mathematical description selected for a physical system completely determines which mathematical apparatus can be used for development of a method of numerical modeling.
4.1 Non-lumped parameters models of a system

Usage of this type of model for interpreting a system as a continuous medium implies application of the mathematical apparatus of mechanics and electrodynamics of continua, hydrodynamics, and perhaps other fields of theoretical physics. The dynamics of a multicomponent physical system is thus described by a set of partial differential equations. The discretization of a continuum model leads to the methods of finite differences, finite elements, boundary elements and other methods usually used in the applicable areas of science. In general, a singular characteristic of three-dimensional MEMS models development is the necessity of accounting for not only the composite geometry of a considered system, but also its non-steady state. The modifications of MEMS topology, characteristic, for example for electrostatic MEMS, are associated with physical processes created in the given topological structures by variations in the outside influences. The mathematical description of this physical problem results in one of the most difficult problems in the field of numerical methods: problems with mobile (free) boundaries [17], inherent not only for Micro-Electro-Mechanical Systems [5, 34–36], but also for other fields of application of simulation methods [43]. The method of MEMS simulation, realized in the simulator AutoMEMS (Coyote Systems Inc.) is described in the paper [25] (see Figure 3). The computational scheme being more or less common to 3D models, well characterizes this class of models. For MEMS simulation in an autonomous mode the following four operations implement the process: the generation of a 3D model on the basis of data about photolithographic masks; the introduction of information about boundary conditions and properties of materials used for construction; generation of a computational grid; the solution of the system of partial differential equations describing the model of the MEMS device. The numerical model introduced in paper [25] is based on the use of the boundary elements method (BEM). Realizations of other methods of MEMS simulation [16, 21, 20] are based on some combination of a finite element method (FEM) and BEM. These methods differ by the various degrees of automation for the described simulation stages.

The paper [5] presents a numerical model of an idealized electrostatically actuated MEMS device. The device consists of an elastic membrane suspended above a rigid plate (Figure 4). The membrane has width \( L \) and is separated from the ground plate by a distance \( l \) in its undeformed state. The model assumes that both membrane and ground plate are infinite in the \( z \) direction. It is also assumed that both membrane and plate are perfect conductors and a potential difference is applied between
them. The equations of the mathematical model associate the electrostatic potential in the region surrounding the device and the elastic displacement of the membrane. The numerical problem of MEMS device simulation is formulated in the paper as a steady-state boundary-value problem.

The governing equations for the electrostatic potential satisfy Laplace’s equation:

\[ \Delta \psi = 0 \]
FIGURE 4: Geometry of electrostatically actuated device [5]

with boundary conditions:

(2) \[ \psi = 0, \quad \text{at} \quad y = -l, \quad -\frac{L}{2} < x < \frac{L}{2} \]

(3) \[ \psi = V, \quad \text{at} \quad y = u, \quad -\frac{L}{2} < x < \frac{L}{2} \]

where \(V\) is the applied voltage and \(u(x,t)\) is the displacement of the membrane from the \(y = 0\) position, \(x\) and \(y\) are coordinates, \(t\) is time. In the mathematical model the displacement of the membrane satisfies the following equation:

(4) \[ \rho \frac{\partial^2 u}{\partial t^2} + \nu \frac{\partial u}{\partial t} - T \frac{\partial^2 u}{\partial x^2} = \frac{|E|^2}{8\pi}. \]

Here \(\rho\) is the linear mass density of the membrane, \(\nu\) is a damping coefficient, \(T\) is tension and \(E\) is the electric field evaluated along the membrane surface. The first term on the left side of equation (4) is associated with the inertia of the moving membrane. The second term describes the force of mechanical resistance to motion of the membrane. The third term approximates the force arising in the membrane due its elastic strain. The term on the right side of this equation is associated...
with the force of electric attraction acting on the membrane. Equation (4) must be solved with the following boundary conditions:

\[ u \left( -\frac{L}{2}, t \right) = u \left( \frac{L}{2}, t \right) = 0. \]

This equation describes the type of fastening of the membrane edges. It is necessary to note that equation (4) has both steady-state and non-steady-state solutions. For a numerical solution one can set time derivatives equal to zero on the left side of (4). In this case it will be possible to investigate only steady-state solutions of this MEMS simulation problem. Another method of solution is to use iteration methods. In this case it is possible to solve both steady-state and transient problems. This works because the characteristic time for electric field stabilization is much less than for membrane motion. A method for quasi-three-dimensional numerical simulation of Micro-Electro-Mechanical Systems with fixed-fixed beam geometry is developed in the paper [44]. The mathematical problem formulation is based on geometric domain decomposition into two parts: elastic beam and air gap domains (Figure 5).

The state of this system is described by the Laplace equation for this MEMS model:

\[ \Delta \psi_1 = 0 \]

\[ \nabla (\epsilon_p \nabla \psi_2) = 0 \]

where \( \epsilon_p \) is dielectric permittivity of plate material, \( \psi_2(x, z, t) \) is the electric potential in the elastic beam, \( \psi_1(x, z, t) \) is the electric potential in the air gap. A method of mathematical description for fixed-fixed beam movement depending on the voltage applied between the beam and substrate is proposed in a paper in preparation. The equation of plate motion is introduced in this paper under the assumption of shear deformation of the elastic beam:

\[ m \ddot{a} = \ddot{F}_d + \ddot{F}_s + \ddot{F}_e \]

where \( m \) is the mass of the beam element, \( a \) is the acceleration of the beam element, \( F_d \) is the force of viscous damping, \( F_s \) is the elastic force acting on the element, \( F_e \) is the force of the electric field on the element. The resulting plate shape dynamics has the following mathematical formulation:
FIGURE 5: Geometric domain of numerical problem solution at electrostatic MEMS simulation. Here \( l \) is the depth of air gap in the absence of a potential difference between the beam and substrate, \( d \) is the half-thickness of the elastic beam, \( u(x, t) \) is the mid-line of the beam, so that

\[
u(0, t) = l + d
\]

(9) \[
\Delta_{tt} u + \frac{D_e}{d_d} \cdot \frac{1}{1 + (\nabla_x u)^2} \times \left( \frac{\chi_{d1}}{(1 + (\nabla_x u)^2)^{1/2}} (\nabla_t u)^2 + \chi_{d2} (\nabla_x u)^2 \nabla_t u \right)
\]

\[
= G_a \cdot \frac{1}{1 + (\nabla_x u)^2} \left( \frac{\chi_{c1}}{(1 + (\nabla_x u)^2)^{1/2}} (\Delta_{xx} u) \right) + \frac{\chi_{c1}}{d_d} (\nabla_x u)^2 + \frac{\chi_s}{2} (\nabla_x u)^2 \frac{\nabla_z \psi}{\psi}
\]

\[
+ E_m \cdot \frac{\chi_{c2}}{d_d} ((\nabla_x \psi)^2 + (\nabla_z \psi)^2)
\]

where \( \chi_{d1} \) and \( \chi_{d2} \) are parameters depending on both the properties of the substance between the moving plate and substrate and the position of the element; \( \chi_{c1}, \chi_s \) are parameters depending on the elastic prop
ties of the beam; $\chi_{e2}$ is a parameter that depends on electric properties of the substance between the beam and substrate; $De$, $Ga$ and $Em$ are the similarity parameters for the problem. Our method for the numerical solution of equations (6), (7) and (9) developed in [44], is based on the Boundary Elements Method. The numerical solution of the electric potential equations and the equation for the function of air gap size is carried out by minimizing of a functional derived on the basis of the Laplace equation:

$$I = \int_V \left( \frac{\alpha}{2} \left( \frac{\partial \psi}{\partial x} \right)^2 + \frac{\beta}{2} \left( \frac{\partial \psi}{\partial z} \right)^2 \right) dV + \int_S \varphi(x, z) \psi dS$$

where $\alpha$ and $\beta$ are certain constants; the function $\varphi$ describes the boundary conditions for the numerical problem; $V$ and $S$ are the volume and the boundary, respectively. The method proposed in [44] is based on the non-lumped parameters model paradigm and may be used both for the investigation of time-dependent processes arising from changes in the governing voltages of the MEMS and for the study of static MEMS characteristics. The use of this method makes it possible to solve the problem of optimizing constructional parameters for electrostatic MEMS. The obvious advantage of simulation methods based on the non-lumped parameter paradigm is a maximum level of correspondence between a MEMS device’s mathematical model and the physical behavior of the described system. The use of the machinery of deformed body mechanics together with the equations of electro- and thermal physics and hydro- and aerodynamics allows the development of numerical methods corresponding to pre-given properties of conservation for the considered system. That is presently not possible for the majority of methods using lumped-parameter models. Another important feature is the continuity of development, appropriate for this branch of MEMS simulation methods. The feasibility of using developments accumulated in the field of physical systems simulation during more than a half-century, allows avoidance of the solution of a series of very hard scientific problems, inherent for numerical problems in various branches of physics. Among deficiencies for this group of methods it is possible to refer, first of all, to the lack of uniformity and universality of the description of a considered physical system. This results in the necessity of using the descriptive apparatus from separate areas of science, which requires a careful study of a physical analog for the considered system. Another current problem is the lack of a capability of macromodels development, generically associated with a method of full three-dimensional simulation.
4.2 Lumped-parameters models of a system  The use of lumped-parameters models implies the application of a mathematical apparatus to describe systems which consist of discrete elements. The dynamics of MEMS thus is described by a set of ordinary differential equations. For the discretization of a continuum the same set of methods is utilized as with non-lumped parameters models. The philosophy underlying present methods of MEMS simulation is substantially determined by the specific nature of the problem, for the solution of which an appropriate method will be used. It is common for methods of simulation developed for independent problem solving in some definite area of physics to significantly differ from the methods used for the solution of the same class of problems, originating from research on Micro-Electro-Mechanical Systems. Another reason for these differences is the desire of investigators to find a general-purpose method for the description of all constituents of a multicomponent physical system. Both of these reasons exert a significant influence on the development of methods for 3D-models. One of the methods used for MEMS description within the framework of lumped-parameters models is a method universally describing a multicomponent system, based on a formalism of Lagrange \cite{11}. This approach, in the fundamentals of which lies the decomposition of a system into geometrical primitives, uses the apparatus of deformed body dynamics for a description of these elements. This approach is common for fields of construction both of full three-dimensional models and macromodels. The simplest way to integrate simulation tools designed for systems with components of different physical natures can be the selection of one of the components as dominating and the subservience of all the remaining components to laws adopted for the description of this component. This method has naturally appeared in the development of MEMS simulation tools. Another factor moving the development of simulation methods for Micro-Electro-Mechanical Systems in this direction was the high level of efficiency of the tools and methods of electrical circuits simulation. From the idea to join the tools of simulation with the electrical simulator came the method known as Modified Nodal Analysis \cite{9}. Nodal analysis \cite{32} is presently applied to the formulation of the systems of equations for electrical circuits. This system is used in the simulators that are widely known as SPICE. The method of Modified Nodal Analysis extends and supplements a set of arguments used for the characterization of a modeled system. Many MEMS can be described within the framework of
lumped-parameters models by a system of ODEs of the following form:

\[
M \ddot{q} + D \dot{q} + K q = F(t, q, \dot{q})
\]

where \( M, D \) and \( K \) are matrices of masses, friction coefficients and elasticity, respectively. The discretization of the system’s geometry on \( N \) elements results means that the vector of coordinates of elements-nodes \( q \) has dimension \( 6N \), as well as vectors of forces and moments of forces \( F \), acting on elements. The mechanical component of a system has \( 6N \) degrees of freedom [3] and can be described by a system of \( 6N \) equations. Accounting for the electrical degrees of freedom results in an increase in the order of the system of equations. If the number of variables describing the electrical MEMS constituents is equal to \( m \), then the general order of the system of equations describing the MEMS dynamics becomes equal to \( 6 \cdot N_{\text{Mechanical}} + m \cdot N_{\text{Electrical}} \). A similar approach can be used for taking into account components of a physical nature distinct from electrical. The resulting set of equations, based on Newton’s laws, can be considered as a mechanical analog of the Kirchoff equations, using the analogy of current to force, and electric potential of a unit to movement of the MEMS element. The solution of full three-dimensional problems of static analysis and transient analysis in MEMS devices is implemented on the basis of the given information on properties of the materials used (elastic modulus, Poisson’s constant, coefficient of thermal expansion, etc.), geometry of the system, electric potential difference and forces applied to different parts of the system. The method of Modified Nodal Analysis is used by the simulator SUGAR [3, 8], designed in the Sensor and Actuator Center, University of California, Berkeley. Algorithms of a method of solution are given also in the well-known program MATLAB [26]. The systems of equations (10) may be used for the simulation of an RF-switch, shown in Figure 6. For the linear case, the matrices \( M, D \) and \( K \) can be determined as follows:

\[
M = \frac{\rho AL}{420} \cdot \\
\begin{bmatrix}
140 & 0 & 0 & 70 & 0 & 0 \\
0 & 156 & 22L & 0 & 54 & -13L \\
0 & 22L & 4L^2 & 0 & 13L & -3L^2 \\
70 & 0 & 0 & 140 & 0 & 0 \\
0 & 54 & 13L & 0 & 156 & -22L \\
0 & -13L & -3L^2 & 0 & -22L & 4L^2
\end{bmatrix}
\]
where \( E \) is Young modulus; \( L \) is the length of the beam; \( A \) is the cross sectional area of the beam; \( I \) is the moment of inertia; \( \rho \) is the density of beam material; \( \mu \) is the viscosity of the substance between the beam and substrate; \( w \) is the width of the beam; \( \delta \) is the distance from the beam to substrate. Modified Nodal Analysis underlies the simulator NODAS [47], designed at Carnegie Mellon University. There are characteristic problems with approximation of continuous physical phenomena by discrete relations. In particular, the problems of approximation of forces damping mechanical motion create some of the major difficulties. Another idea of adjoining the tools of MEMS simulation to electrical simulators was the direction based on development of Equivalent Cir-
cuit Models. The method of electrical analogies in its fundamentals is widely applied in different areas of science and engineering, among them mechanics, thermal physics, acoustics and the field of Micro-Electro-Mechanical Systems [48, 49]. It is based on the mathematical analogy between dynamical equations of a considered system and some electrical circuit (see Table 1).

<table>
<thead>
<tr>
<th>Mechanical parameter</th>
<th>Electrical parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity, angular velocity</td>
<td>Voltage</td>
</tr>
<tr>
<td>Force, torque</td>
<td>Current</td>
</tr>
<tr>
<td>Damping</td>
<td>Conductance</td>
</tr>
<tr>
<td>Compliance</td>
<td>Inductance</td>
</tr>
<tr>
<td>Mass, mass moment of inertia</td>
<td>Capacitance</td>
</tr>
<tr>
<td>Compliance per unit length</td>
<td>Inductance per unit length</td>
</tr>
<tr>
<td>Mass per unit length</td>
<td>Capacitance per unit length</td>
</tr>
<tr>
<td>Characteristic mobility</td>
<td>Characteristic impedance</td>
</tr>
<tr>
<td>Mobility</td>
<td>Impedance</td>
</tr>
<tr>
<td>Impedance</td>
<td>Admittance</td>
</tr>
<tr>
<td>Clamped point</td>
<td>Short circuit</td>
</tr>
<tr>
<td>Free point</td>
<td>Open circuit</td>
</tr>
<tr>
<td>Force</td>
<td>Current</td>
</tr>
<tr>
<td>Velocity</td>
<td>Voltage</td>
</tr>
</tbody>
</table>

**TABLE 1: Electromechanical mobility analogies**

The similarity of the equations implies the mathematical equivalence of physical quantities from different areas of physics. A fundamental problem in this approach is the search for the electrical circuit which would be described by equations similar to the dynamical equations of an investigated Micro-Electro-Mechanical System. One of most typical applications of the electrical analogies method is the simulator APLAC of the company Aplac Solutions, which produces a realization of a simulation capability for MEMS, significantly different at the level of a physical analog. The papers [50, 54] were the ideological basis for the use of the Equivalent Circuit Model for the simulation of multicomponent systems. There it was shown that electrical, fluid, mechanical and thermal sub-systems can be jointly described on the basis of the Kirchoff formalism applied to electric networks. It was a natural development, realized in the simulator APLAC, to construct a library of equivalent networks intended for the simulation of electromechanical and fluid MEMS components. The most attractive property of this method is the fact that as the MEMS models are described by electrical equivalent networks,
the description of MEMS becomes homogeneous. An equivalent network for an electro-mechanical system of capacitive type is shown in Figure 7. The model of mass suspended on a spring [50, 51] describes this type of devices quite well. The spring is modeled as gyrator and capacitance $C_z$. The capacitance $C$ is the equivalent of mass. Damping in the Micro-Electro-Mechanical System is modeled by conductance $G$.

![Equivalent network of MEMS](image)

**FIGURE 7:** The equivalent network of MEMS that is described as a mass-spring system [48]

Figure 8 presents the structure and operating principles of a capacitive MEMS switch [53]. The bridge is perforated to reduce the damping of air flow.

The differential equation describing the motion of the bridge has the following form:

$$m \frac{\partial^2 z}{\partial t^2} + \gamma \frac{\partial z}{\partial t} + EI_y \frac{\partial^4 z}{\partial x^4} + hwS \frac{\partial^2 z}{\partial x^2} = f(x, z)$$

where $m$ is the mass of the bridge, $z$ is the displacement, $l$ is the coordinate along the beam, $\gamma$ is the damping term due to the gas flow, $E$ is the effective Youngs modulus, $I$ is the moment of inertia, $S$ is the stress of the beam, $h$ is the height of the beam, $w$ is the width of the bridge, $f$ is the distributed load due to electrostatic actuating force and the mechanical contact force. This model of an RF MEM switch was constructed from elementary finite difference sections. These sections consist of elements that model bridge deflection, electrostatic actuation force, gap capacitance and mechanical contact force. The construction...
The two first and two last sections model the clamped-clamped boundary conditions. All other model sections are identical.

Each section contains a model of beam deflection (BE), mechanical contact (NC), gas damper (PGD) and electromechanical transducer (NTR) with electrostatic actuation force and gap capacitance. The capacitances of sections are connected in parallel. The paper [53] presents in addition a calculation method for the air flow through the perforation holes. Numerical modeling of the micro-electro-mechanical component functioning of a system can be performed together with the model for networks of electronic control both in frequency and in time domains [48, 50]. The method of electrical equivalent networks has obvious advantages, but also has deficiencies. One is associated with the precision of the switch model is shown in Figure 9.
of approximation in the case of essentially nonlinear systems [52]. In particular, the models used in microfluidic MEM subsystems approximate the process of gas flow well enough only in the case when the gas pressure and change in position of MEMS plate are small enough in comparison with static values.

5 Conclusion Modern methods of MEMS simulation have passed through a long period of development. They have been realized through such tools as MEMCAD (M.I.T.), CAEMEMS (University of Michigan), CAPSIM (Catholic University of Leuven), SENSOR (Fraunhofer Institute), SESES (ETH), IntellICAD (IntelliSense Corp.), CoventorWare (Coventor Inc.), APLAC (Aplac Solutions), NODAS (Carnegie Mellon University), SUGAR (University of California at Berkeley) and many others. And the problems of MEMS simulation methods design have not lost importance since the beginning of the 90s because of the ar-
rival of new material technologies and new devices types. The solution of problems of miniaturization for the field of Micro-Electro-Mechanical Systems results in an increase in financial support for the development of nano-size devices [1, 2, 4]. The result is the current appearance of devices, the principle of operation of which is based on effects from quantum physics. Independent progress in different areas of nanoscopic systems, for example in nano-electro-mechanical systems (NEMS), single-electron devices (SED), nano-fluidic systems (NFC), will in several years lead to the creation of new areas of science and engineering. Not only the development of the design tools for these devices, but also the methods of their simulation, will differ considerably from ones existing now. One of the features of models of nanoscopic devices is that the description of a system must be done at the molecular and atomic level. The numerical effectiveness of methods of simulation for these systems is in many respects determined by the effectiveness of their description by means of stochastic processes. Different levels of physical models used for the study of dynamic phenomena in nanoscopic system will already in the near future create the necessity of the development of approaches using methods of molecular dynamics as a base for the numerical methods, and the theory of control of stochastic systems as a fundamental for the construction of automated design systems of a new generation—CAD of nano-electro-mechanical systems.

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