LOCATING ANOMALOUS SEISMIC ATTENUATION: A MATHEMATICAL INVESTIGATION

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1 Introduction  Seismic imaging, a technique in which the reflections of a source seismic wave are recorded as it passes through the earth, is a major tool for geophysical exploration. A detonating charge creates a seismic wave which propagates through the earth, and surface geophones record the reflections of this wave from different geological layers. Seismic imaging can be used to reconstruct a profile of the material properties of the earth below the surface, and is thus widely used for locating hydrocarbons.

In this paper we are concerned with the detection of anomalous seismic attenuation: the loss of energy of a seismic wave as it propagates through the earth. Specifically, attenuation is defined as the loss of energy of a seismic wave as it travels though the earth, which is not caused by geometric spreading, but depends on the characteristics of the transmitting media. As an exploration tool, attenuation effects have only recently attracted attention. These effects can prove useful in two ways: as a means of correcting seismic data to enhance resolution of standard imaging techniques, and as a direct hydrocarbon indicator. Many physical processes can lead to the attenuation of a seismic trace. These can

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be divided into two main categories: scattering and absorption, depending on the way energy is transformed. Scattering attenuation occurs because particles in the earth redirect the sound wave in other directions and happens when “the scale of heterogeneities is smaller than the characteristic wavelength of the seismic wave” [19]. The energy is not dissipated, merely redirected. On the other hand, intrinsic attenuation is caused by absorption, or the conversion of acoustic energy into thermal energy. In other words, intrinsic attenuation is mainly due to friction losses. Absorption and scattering effects are difficult to separate; we assume most of the loss in energy is caused by intrinsic attenuation and focus on these effects exclusively.

Intrinsic attenuation is caused by friction, particularly in porous rocks between fluid and solid particles, (see [7, 2]). Though there is no agreement on the mechanism involved, it has been observed that porous rocks filled with fluid considerably attenuate a seismic signal, with the attenuation being strongest in a partially fluid-saturated rock [17, 18, 22]. Attenuation varies with many factors such as water saturation, porosity, pore geometry and pressure.

This work is motivated by a problem originally posed by Husky Energy at the 8th Industrial Problem Solving Workshop organized by the Pacific Institute of Mathematical Sciences, May 2004. The goal of the workshop was to find a means of computing seismic attenuation from relatively short windows of seismic imaging data, and particularly to be able to identify regions of anomalous attenuation.

The paper is organized as follows. We begin by a detailed description of the attenuation problem in Section 2, collecting important notation and assumptions for easy reference.

In Section 3, we consider the use of frequency-shift techniques to identify anomalous attenuation; several different attributes are tested on real and synthetic data. In Section 4, we present the mathematical ideas behind an extension of a Wiener technique. In Section 5, we describe some wavelet-based strategies, and in Section 6 we sketch an optimization-based algorithm. We end the paper in Section 7 with ideas for future work.

Please note that captions and discussions regarding figures are made to color values. For a full-color version of this paper, please see the CAMQ website at

http://www.math.ualberta.ca/ami/CAMQ/online.htm
2 Description of problem  The attenuation of the wave is directly linked to the different layers that compose the Earth, so that whenever changes in the composition of layers occur, the attenuation changes too. This is why we would like to be able to detect changes in attenuation, which would enable us to identify changes in material properties.

In this section, we begin by defining some notation. We then present a problem statement, and end with a description of the data we will be working with.

2.1 Notations and definitions  A detonation creates the source waveform we will usually call $w(t)$, which propagates through the earth. This may also be called the “signature of the source.” For a finite time, the reflected signal is recorded by a receiver at discrete time intervals. This signal, $s(t)$, is called a seismic trace. If we assume the propagation speed, $c$, to be constant, the time $t_n$ will correspond roughly to reflections from depth $(ct_n)/2$. Since the waveform $w(t)$ is caused by a detonation, it is not a delta pulse, but stretches over a finite time interval; therefore reflections from layers $n-1$, and perhaps $n-2$, etc. will also contribute to the signal at $t_n$. Furthermore, the coefficients of reflection from different layers in the earth are unknown, as is $w(t)$. Several source-receiver pairs are located along a line and a trace is recorded at each one. The collection of traces is called a seismic section.

It might be useful to note that in the literature the models for seismic wave propagation are often illustrated using specific methods of collecting seismic data, which are different from surface reflection profiles (e.g., convolutional model in [23], p. 162). A vertical seismic profile (VSP) refers to measurements made with a source at the surface and many receivers positioned down a vertical well. The receivers record full traces. Sonic logs are vertical seismic profiles where only the arrival time, not the full trace, is recorded at sparsely-spaced geophones. In both cases the velocity in the vertical direction is measured, and changes in velocity are related to changes in geological layers [12]. These strategies are expensive, since they require the drilling of wells. As a preliminary step in exploration, it is cheaper to collect reflection information at the surface. For this reason, we are interested in surface seismic reflection profiles, where we have assumed the propagation speed of the wave to be constant. Although this will affect resolution, we can still obtain information about seismic boundaries.

The ability of a material to attenuate seismic waves is measured by a dimensionless quantity $Q$, called the attenuation factor, which describes the energy loss of a seismic wave per cycle of oscillation. We follow one
pulse through the medium, and observe how it’s energy decays over a wavelength. Specifically, the attenuation factor is defined by

\[
Q := \frac{\text{energy of seismic wave}}{\text{energy dissipated per cycle of wave}} = \frac{2\pi E}{-\Delta E}
\]

where \(E\) is the energy of the wave, and \(\Delta E\) is the change in energy per cycle. Thus, lower \(Q\) values are related to high energy loss. Typical values of \(Q\) range from 5–20 (dirt) through 100 (rock) to 10,000 (steel). The reservoirs of interest, bearing hydrocarbons in fluid or gas form, typically have a \(Q\) value of between 20 and 40, and are more attenuative than the surrounding rocks. In what follows, we assume that this attenuation factor is independent of frequency \(\omega\) in the useful seismic bandwidth, but that it changes with depth.

2.2 Problem statement The goal of the work conducted at the IPSW, and in this paper, was to locate efficient methods for computing changes in \(Q\) values from given seismic trace data. In particular, we are not interested in the actual \(Q\) values, only in being able to identify regions in space where these values change anomalously. Since we assume the speed of the wave is constant, this becomes equivalent to locating temporal regions where the values change dramatically.

Our strategy was to develop algorithms to detect anomalous attenuation, and then test these methods on simulated data as well as real seismic data, where the location of the anomaly is known. This helps us compare the effectiveness of different strategies, at least on these data sets.

2.3 Description of data The data consists of surface seismic reflection profiles. The experimental set-up is assumed to be one dimensional. That is, all geological layers are horizontal, and the signal only travels along the vertical direction. We also assume that source and receiver geophones are coincident and positioned directly on the surface of the earth (zero offset). The source emits a pulse at time \(t = 0\), and the geophones immediately begin collecting data.

Two real data sets were provided during the IPSW workshop. The first comes from Pike Peaks field, discovered in 1970 in east-central Saskatchewan, where the hydrocarbon production is from the lower Cretaceous Waseca formation that is predominantly quartz, well-sorted channel sand of 33% porosity. It is about 500 m deep and 10 to 30 m thick. The oil is 12 degree API with a GOR of 15,\(^{[14]}\). The seismic section consists of 761 traces. Each trace was sampled for 1 second, at
intervals of time 0.002 s. The reservoir is located at around trace 400, and depth corresponding to a time of about 0.7 seconds.

The second data set is of the Blackfoot reservoir located southeast of Calgary in Alberta. The reservoir is a channel filled with porous cemented sand at 1550 m below the surface and thickness of 10 to 20 m. The seismic section consists of 86 traces. Each trace was sampled for 2.7 seconds, at time intervals of 0.002 s. The reservoir is located at traces 40–50, and depth corresponding to a time of about 1.2 s.

Figure 1 shows the seismic sections and the traces from each data set.

The data sets we have received have undergone some processing related to the data acquisition. We do not know the precise nature of this processing. For example, a possible step might have been to perform the experiment with different sources at the same location, and then to average them into a single seismic profile. The two sets of data may have undergone different pre-processing corrections, especially since the data is also used in other contexts, not just attenuation measurement. In particular, some filtering of higher-frequency modes may have been done; since attenuation is best measured in this end of the spectrum, the preprocessing rendered the problem quite challenging.

Starting from the data received, our goal is to identify the reservoir locations, by locating regions of anomalous attenuation.

In order to test ideas, we also generated some synthetic data where the attenuation anomaly is known. Two traces are produced. Each is sampled for \( t = 2 \) s, at intervals of time 0.002 s. The reflection coefficients are drawn from a random Gaussian noise distribution and then raised to an integral power, making the sequence more spiky. The phase velocity is assumed to be normalized so that \( c = 1 \) and the source signal \( w \) has a dominant frequency of 20 Hz. The two traces differ only in the attenuation coefficients of the material of propagation:

- One has a constant attenuation coefficient \( Q = 100 \).
- The other has a \( Q = 20 \) at a depth \( ct = 1 \) until \( ct = 1.1 \) (where \( c = 1 \)), and \( Q = 100 \) everywhere else. This simulates a hydrocarbon reservoir of depth 0.1.

The two seismic traces \( s(t) \) are calculated as they propagate. They appear nearly identical in time, but if we take their difference we can locate the onset of the anomaly. Figure 2 shows the simulated data.

3 Frequency shift methods for attenuation detection

A popular strategy for detecting anomalous attenuation is to study how the
(a) Pikes Peak: anomaly at depth 350, location x=400.

(b) Blackfoot reservoir: anomaly at depth t=600, location x=40–50.

FIGURE 1: Actual seismic traces: Pikes Peak and Blackfoot.

frequency profile of a pulse changes as it passes through the earth. We will see next that higher frequencies are attenuated more than lower ones; the average frequency of the pulse will therefore decrease as it is attenuated. Our strategy in this section is to develop frequency-based “attributes” which are easy to compute, and which highlight abnormal
(a) Profile of “normal” and “anomalous” attenuation.

(b) l-r: normal trace, anomalous trace, traces superimposed, difference of traces

FIGURE 2: The seismic trace corresponding to the attenuation anomaly is nearly identical to the normal attenuation case.
shifts in the frequency best.

Suppose the signal observed is of form $s(t) = A(\omega, t) \cos(v(t))$, where $A(\omega, t)$ is the amplitude modulation as a function of frequency $\omega$ and time $t$. For a medium with linear stress-strain relation, it is known that the wave amplitude $A$ is proportional to $\sqrt{E}$, where $E$ is the energy of the wave. Hence, the attenuation factor, which depends on the depth, is

\begin{equation}
Q(z) = \frac{2\pi (A(z))^2}{-2A(z) \Delta A} \rightarrow \frac{1}{Q(z)} = -\frac{\Delta A}{\pi A}
\end{equation}

from which we can obtain the amplitude changes due to attenuation. That is, given the initial amplitude $A_0(\omega, t)$, let $\lambda$ be the wave length given in terms of frequency $\omega$ and phase velocity $c$ by $\lambda = 2\pi c/\omega$, then $\Delta A = \lambda (dA/dz)$. Equation (2) becomes

\begin{equation}
\frac{dA}{dz} = -\frac{\omega}{2cQ(z)} A(\omega, z)
\end{equation}

with the exponential decaying solution

\begin{equation}
A(\omega, z) = A_0(\omega, t) \exp \left( -\frac{\omega z}{2cQ(z)} \right).
\end{equation}

Now, from observation of exponentially decaying values of $A(\omega, z)$, we can compute the $Q(z)$ value. That is, from (4), we have

\begin{equation}
\ln \left( \frac{A(\omega, t)}{A_0(\omega, t)} \right) = -\omega \left( \frac{z}{2cQ(z)} \right) = -\omega \left( \frac{t}{2Q(z)} \right).
\end{equation}

Here we assume that the phase velocity $c$ does not depend on frequency, i.e., that there are no dispersion effects. This has the added effect of correlating well the time of travel of the reflected wave with the depth of the layer from which the reflection occurs. Hence, by recording the $\ln(A/A_0)$ versus $\omega$ graph, and then estimating the average slope, we can recover the value of $Q(z)$ at the given depth. This idea is known as log spectral ratio method. Even though attenuation decreases both the amplitude as well as the average frequency of a seismic wave as it propagates, amplitude decay is more likely to be affected by noise. Hence the frequency shift is a more reliable method of estimating the attenuation [19]. We will work with the frequency representation of the signal, and wish to target anomalous high frequency energy loss.
For seismic imaging we can deduce a spectral ratio formula which includes the effect of reflections from discrete geological layers. Let the reflection coefficient of the $n^{th}$ layer be $r_n$. Assume a temporally localized seismic pulse $w(t)$ is the source seismic wave, which propagates through the earth, being reflected and attenuated over time. The seismic traces $s(t)$ is collected at times $t_1, t_2, \ldots, t_n$. Suppose we hear the echo around times $t_m, t_{m+k}, t_{m+2k}$, etc. These need not be successive time intervals. In the Fourier domain at frequency $\omega$, we may write

$$\left| \hat{s}_k(\omega) \right| = r_k \left| \hat{s}_o(\omega) \right| \exp\left(-p\omega t_k/Q_k\right)$$

where $p$ is a constant, and $r_k$ is the coefficient of reflectivity of the $k^{th}$ layer.

Suppose we have similar information about a seismic trace reflected from layer $j$, and that the attenuation factor $Q_k$ has not changed between the $k^{th}$ and $j^{th}$ layer. Then the log spectral ratio method estimates the attenuation $Q_k$ as

$$\log \left( \frac{\left| \hat{s}_k(\omega) \right|}{\left| \hat{s}_j(\omega) \right|} \right) = \log |r_k| - \log |r_j| + \frac{p\omega(t_j - t_k)}{Q_k}.$$  

To use the log spectral ratio method, we divide an observed trace into several “windows” comprised of a few layers in the temporal direction, and assume the attenuation $Q$ is constant over each of these. We then compute the Fourier transform of each of these windows, and use the log spectral ratio to compute the attenuation factor in each. Variations in this factor across windows should highlight geophysical variations. Clearly, we would wish for the windows to be narrow so as to more precisely identify the location of the anomaly. Unfortunately, this windowing procedure contaminates the Fourier transform of the signal. To alleviate this problem, we could also take a Gabor transform of the seismic trace at each of these times. Then we get

$$|\hat{s}_g(\omega, t_m)| = r_m |\hat{w}(\omega)| \exp\left(-p\omega t_m/Q\right),$$

where $p$ is a constant and $\hat{s}_g(\omega, t_m)$ is the amplitude of the frequency $\omega$ in the Gabor-transformed signal at time $t_m$. The same is true for $|\hat{s}(\omega, t_{m+k})|$. The log spectral ratio estimates $Q$ as:

$$\log \left( \frac{|\hat{s}_g(\omega, t_{m+k})|}{|\hat{s}_g(\omega, t_m)|} \right) = \log |r_{m+k}| - \log |r_m| + \frac{p\omega(t_{m+k} - t_m)}{Q}.$$
3.1 Centroidal frequency Since the seismic traces are noisy, calculation based on individual frequencies is not robust. However, we expect to observe a downshift in the average of the signal’s frequency spectrum, since the high frequency components are attenuated more rapidly than the low frequency components. To find how the centroidal frequency of the signal \( s(t) \) changes as it propagates in the earth, we first choose a (large) number of points in time, \( t_0, t_1, \ldots, t_n \) and a fixed window length. Successive windows should overlap. At each of the points \( t_j \), we calculate the Gabor transform of the windowed signal, then the centroid frequency for that window:

\[
f_c(t_j) = \frac{\int \omega |\tilde{s}_g(\omega, t_j)| d\omega}{\int |\tilde{s}_g(\omega, t_j)| d\omega} \approx \frac{\sum_k \omega_k |\tilde{s}_g(\omega_k, t_j)|}{\sum_k |\tilde{s}_g(\omega_k, t_j)|},
\]

and also the second moment:

\[
f_c(t_j) = \frac{\int \omega^2 |\tilde{s}_g(\omega, t_j)| d\omega}{\int |\tilde{s}_g(\omega, t_j)| d\omega} \approx \frac{\sum_k \omega_k^2 |\tilde{s}_g(\omega_k, t_j)|}{\sum_k |\tilde{s}_g(\omega_k, t_j)|}.
\]

The centroidal frequency and the second moment are two possible “attributes” of the signal which we can compute with the aim of identifying an anomalous attenuation of the signal.

We tested the centroid method on both the synthetic and the real data. For the synthetic case, we clearly see a down shift in the centroid frequency in the abnormal trace at 1 s, the onset of the abnormality, in comparison with the normal trace (Figure 3). With the real data we have no ‘normal’ trace for comparison, yet there are regions of sudden drop in the centroid frequency at the regions where the reservoirs are located. In the figures, blue indicates lower frequency and red indicates higher frequency. For the Pike Peaks data in the centroid figure and even more clearly in the second moment figure there is a spot of blue at \( t = 0.7 \) s, traces 350 to 400, (Figure 4). Similarly, for the Blackfoot data (Figure 5), there is a region of cyan between traces 40–50, at time of about 1.2 s corresponding to the location of the reservoirs. There is a region of blue at the same location for the second moment, though in this case the area of low frequency is less sharply resolved.

The denominator (the integral of the Gabor transform in each window) seemed to be a good indicator of the onset of the anomaly for the synthetic data, though not for the real data. We therefore do not recommend it as an attribute to track for the purpose of anomaly detection.

The \( \tilde{s}_g(\omega, t_j) \) needed in the calculation of the centroid may be smoothed using a convolution with boxcar function \( b \) of frequency. If \( b(z) \)
is centered around $z = 0$ and falls away far from zero then, similar to the Gabor transform, the function $\hat{s}(u)$ near $u = t$ is emphasized, while oscillatory behaviour far away is dampened. However, the results do not seem much different when the smoothing is applied. For the spectral ratio method, we take the logarithm of the ratio of the Gabor transform of successive windows. Plotted with respect to time the slope, the graph should be linear with slope $1/Q$, if the attenuation factor $Q$ remains constant through the earth. However, noise makes the graph very oscillatory, and often the calculated slope is near to zero, or even negative. We relaxed this method by first trying to fit the logarithm of the ratio $|\hat{s}_g(\omega_1, t_m)|$ with quadratic or cubic polynomials to capture more of the changes.

In the next subsection we investigate some other possible attributes.

3.2 Polynomial fit to spectral ratio The frequency spectrum of each Gabor window was fitted with a polynomial of the desired degree, using a least-squares fit. We store only the leading order coefficient of this polynomial, and track it from window to window. Hence, for a window at time $t_n$ we have a leading coefficient $a_n$. Either $a_n$ was stored for window $n$, or $a_n - a_{n-1}$ was stored. Subtracting successive coefficients
was thought to highlight local changes. We anticipate a steady decrease in high frequency components from the propagation of the wave through the rock, but a sharp drop at the location of the reservoir. Comparing successive windows would compensate for the background decay, accentuating the sharp drop. However, the leading coefficient $a_n$ proved to be a better measure, as opposed to the variation $a_n - a_{n-1}$.

For the synthetic data, the coefficients change at the onset of the abnormality for the linear and quadratic terms when only the leading coefficients are stored, though the change is not large (Figure 6). For the Pike Peaks data, the best results were obtained using a quadratic fit. The area of greatest contrast in the leading order coefficient seems to be around trace 350 and 0.6 seconds (aim: trace 400, time 0.7 s). For the Blackfoot reservoir, the cubic coefficients show a clear low point between trace 45 and 65 and at time 1.2 s, the location of the reservoir. (Figures 7 and 8).

4 Convolutional models and modifications of the Wiener transform method  As a seismic signal propagates through the earth, it is both reflected and attenuated. Mathematically, the observed wave-
(b) Centroid frequency.

(c) Second moment frequency.

FIGURE 4: (contd.) The frequency method applied to the Pike Peaks data. The centroid and second moments method clearly show a blue region at trace 400, depth corresponding to 0.7 s. This low point corresponds to a downshift in the average frequency due to high attenuation.
form is a convolution between the source and the reflectivity coefficients (in the absence of attenuation). The effect of attenuation is mathematically modeled by the action of a pseudo-differential operator on the source signal. In this section, we study these convolutional models, and seek to use them to identify anomalous attenuation.

### 4.1 Convolutional model without attenuation

We will begin with the model without attenuation. Our discussion is based on [23], and the lecture notes of Tim Henstock [10], and is included here for completeness. Let

\[ s(t) = \text{unattenuated seismic trace received at time } t \text{ at a receiver}, \]
\[ w(\tau) = \text{source waveform or signature}, \]
\[ r(t) = \text{reflectivity as a function of depth } (\equiv \text{time}). \]

Let us recall two of the assumptions mentioned briefly in the data section. We assume that:

1. The earth consists of stacks of horizontal layers, with the same composition in each layer.

![Regular integral](image)

(a) Integral of the Gabor transform (denominator).

**FIGURE 5:** The frequency method applied to the Blackfoot data. The centroid graph shows a low region at 1.2 s, traces 45-65 which is near location of the reservoir at traces 40-50.
FIGURE 5: (contd.) The frequency method applied to the Blackfoot data. The centroid graph shows a low region at 1.2 s, traces 45–65 which is near location of the reservoir at traces 40–50. For the second moment, the low area is less sharply resolved.
(a) Leading coefficients

(b) Subtract leading coefficients of consecutive windows

FIGURE 6: Coefficients of the linear, quadratic, and cubic polynomials fit to the absolute Gabor transform over many windows for the synthetic trace.
FIGURE 7: The polynomial fit of the Pike Peaks data. The Gabor transform was taken for successive windows, and fitted with a polynomial for each window. This graph represents the leading coefficients. The abnormal areas seem to be at trace 350 and 0.6 seconds, instead of at trace 400 and 0.7 seconds. However, the quadratic coefficients plot shows a high (dark red region) at the required location. From the previous graph, we expect the quadratic coefficients to give the best results.
2. The source generates a compressional wave (no shear wave) which travels only in the vertical direction. That is, all the incident and reflected waves are normal to the horizontal interfaces.

The combination of these two assumptions makes the seismic trace one dimensional. Attenuation and dispersion cause an amplitude decay in the source wave as it travels through the earth. The time-dependent change in the waveform is called nonstationarity. For the simple convolutional model we make a third assumption.

3. The source waveform does not change as it propagates through the earth.

From assumption (1), the source wave will only be reflected at the boundary between two layers of different materials and the amplitude
of the reflection is dependent on the properties of the materials. So in
this simple model, the sequence of reflectivity coefficients corresponds
to the earth’s impulse response. Reflectivity is measured as a function
of two-way time travel to that interface, where the time corresponds to
depth.

Suppose we have a reflectivity sequence \( r(t) \) and a source wave \( w(\tau) \),
and a trace recorded at \( t_0 \). If \( w(\tau) \) is a simple spike (a delta function),
then we simply have

\[
  s(t_0) = r(t_0).
\]

However, a detonation, which is the typical initial signal, has a short,
but finite width. For a finite width pulse, only the beginning part of
the wavelet, \( w(0) \), will reach the reflection boundary and return to the
surface at \( t_0 \). But the parts of the pulse at \( \tau > 0 \) have not had time to
reach the reflection layer \( r(t_0) \), so that the contribution from \( \tau \) within
the wavelet is \( w(\tau)r(t_0 - \tau) \). The total contribution is the integral

\[
  \int_0^t w(\tau)r(t_0 - \tau) d\tau.
\]

FIGURE 8: The polynomial fit for the Blackfoot data. In this case the
quadratic coefficients are not a good indicator of the reservoir location.
However, for the cubic coefficients graph, a dark blue region appears at
trace 45–65 and time 1.2 s, at the location of the reservoir. This region
is well resolved, similar to the centroid method result.
FIGURE 8: (contd.) The polynomial fit for the Blackfoot data. In this case the quadratic coefficients are not a good indicator of the reservoir location. However, for the cubic coefficients graph, a dark blue region appears at trace 45-65 and time 1.2 s, at the location of the reservoir. This region is well resolved, similar to the centroid method result.
of these:
\[ s(t_0) = \int_{-\infty}^{\infty} w(\tau) r(t_0 - \tau) d\tau, \]

or
\[ s(t) = w(t) * r(t), \]

a convolution. Our (restated) goal is to estimate the earth’s response, or \( r(t) \), but even with this simple model it is difficult to identify closely spaced reflecting boundaries given only \( s(t) \).

To summarize, for the model without attenuation we have
\[ s(t) = w(t) * r(t) + n(t), \]

where \( n(t) \) is noise, which we assume is uncorrelated. External sources
of noise include wind motion, environmental noise and bad coupling of
geophone to the ground, and internal noise from the recording instru-
m ents. We now have one equation, three unknowns \( w(t), r(t) \) and \( n(t) \).
In order to proceed, we make a fourth and fifth assumption:

4. The noise component \( n(t) \) is negligible.
5. We know the source wave \( w(t) \).

The trouble is, we don’t know \( w(t) \). For marine seismic data the
seismic wavelet produced from an air-gun can be estimated from a mea-
 surement near the source, but for detonations \( w(t) \) is unknown. Some
argue that \( w(t) \) should be estimated from source signature measurements
(e.g., [24]), but the common procedure is to obtain \( w(t) \) by working in
the frequency domain and replacing assumption five by

5’.The earth’s reflectivity response has a white spectrum. The spec-
trum is constant amplitude where all frequencies are present. This is
satisfied by a random sampling from a Gaussian distribution.

The fifth assumption is justified because \( \hat{s} (\omega) \), the spectrum of the
signal in the frequency domain has a uniform background, but the basic
shape resembles the spectrum of the source wave. The small oscillations
are due to the reflectivity.

The white reflectivity assumption means that
\[ r * \hat{r} = \int_{-\infty}^{\infty} \hat{r}(s) r(t - s) ds = \delta(t) \]
\( \delta(t) \) is the Dirac measure and \( r(s) = r(-s) \). In other words, the autocorrelation function of \( r \) is a delta function. The reflection at some depth is not related to what is reflected at another depth. Given this assumption, we are able to recover the amplitude of the source signal from the trace

\[
(8) \quad s * \tilde{s} = (w * r) * (\tilde{w} * \tilde{r}) = (w * \tilde{w}) * \delta.
\]

For general \( f \), in the Fourier domain,

\[
\begin{align*}
\hat{f}(t) &= f(-t), \\
\hat{f}(\omega) &= \mathcal{F}[f(t)], \\
\hat{f}(\omega) &= \mathcal{F} \left[ \hat{f}(t) \right] = \hat{f}^*(\omega), \\
\mathcal{F} [s(t) * \tilde{s}(t)] &= \mathcal{F} [s(t)] \mathcal{F} [\tilde{s}(t)] = \hat{f}(\omega)\hat{\tilde{f}}^*(\omega) = |\hat{f}(\omega)|^2.
\end{align*}
\]

The superscript * denotes conjugation, while regular * denotes convolution. Taking the Fourier transform of (8), we have that

\[
(9) \quad |\tilde{s}(\omega)|^2 = |\hat{w}(\omega)|^2.
\]

This is an experimental observation as well concerning the source signal and the observed trace. We see that we can arrive at it theoretically if we assume that the reflectivity is a random sequence which has a white spectrum in the frequency domain.

We have the amplitude spectrum (or gain), \(|\hat{w}(\omega)|\), but to get back to \( \hat{w}(\omega) \) and the time representation \( w(t) \), we also require the phase. Many profiles can have the same amplitude spectrum as \( \hat{w}(\omega) \). Our choices reduce to one if we make the following assumption about the phase:

6. The source wavelet is minimum phase.

The definition of minimum phase is as follows: a stable causal sequence is minimum phase if its Laplace Z-transform has no zeros within the unit circle in the Z-plane. If \( w(t) \) is a causal function, then \( w(t) = 0 \) for \( t < 0 \) and \( w(t) \neq 0 \) for some \( t \geq 0 \). Causality makes intuitive sense because physical systems respond to an excitation after that excitation. All the signals we collect are discrete time series. So if we let \( Z = e^{i\omega \Delta t} \),
then we can rewrite the signal in the time and frequency domain as

\[ s(t) = \sum_{k=0}^{n} s_k \delta(t - k\Delta t), \]

\[ \hat{s}(\omega) = \sum_{k=0}^{n} s_k e^{i\omega k\Delta t} = \sum_{k=0}^{n} s_k Z^k, \]

respectively. Then from (9), we have that:

\[ |\hat{s}(\omega)|^2 = |\hat{w}(\omega)|^2 = (s_0 + s_1 Z + \cdots + s_n Z^n)(s_0^* + s_1^* Z^{-1} + \cdots + s_n^* Z^{-n}) \]

\[ = \prod_{k=1}^{n} (Z - Z_k)(Z^{-1} - Z_k^*). \]

The product was factored into \(2n\) roots which occur in pairs of \(Z_k\) and \(1/Z_k^*\). Half of them are in the unit circle and half of them are outside the unit circle. To get back \(\hat{w}(\omega) \approx \hat{s}(\omega)\), we have to choose \(n\) of these roots. We do not know which ones to choose. This is what is meant when we say we have lost information about the phase. The minimum phase wavelet has all of its roots outside the unit circle. Since \(n\) out of the \(2n\) are outside the unit circle, there is only one wavelet defined by these roots. Another definition of minimum phase is that the wave has all of the energy concentrated at its onset, or has the least energy delay. For detonation sources, this assumption is pretty good, because most of the energy will be concentrated at the front of the wavelet produced. This means that the front end of the seismic wavelet gives the travel time from the source to the reflector and back to the receiver.

The assumptions above mean that the source signal is causal, invertible, and possesses minimum phase in the sense that if we write the signal in the frequency domain

\[ \hat{w}(\omega) = A_0(\omega) \exp(i\phi(\omega)), \]

we can find the phase \(\phi(\omega)\) by using a Hilbert transform. With these assumptions, and in the absence of attenuation, we could recover the source signal \(w\) from a given trace \(s\) using the Wiener process: We already know the amplitude of the source signal \(\hat{w}\) from equation (8) and equation (10), with the minimum phase assumption allows us to retrieve the phase of \(w\).
Unfortunately, the Wiener process is not so simple in the presence of attenuation. Our attempt in the next subsection is to extend this method to the case with attenuation via a perturbation technique.

### 4.2 A convolutional model with attenuation

As mentioned before, Wiener process does not simplify so easily in the case when the signal is attenuated. The following Wiener model with attenuation was originally proposed by Tobias Schaefer \[15\].

Since Assumption 3 from the previous subsection no longer applies, the attenuated trace is now the result of a modified convolution

\[
s(t) = w(t) * a \star r(t) := \int w_\alpha(\tau, t - \tau) r(\tau) d\tau
\]

where

\[
w_\alpha(u, v) = \int \hat{w}(\eta) \alpha(u, \eta) e^{iuv} d\eta,
\]

\[
\alpha(u, \eta) = \exp \left( -\frac{\eta u}{2Q} \right) \exp \left( \frac{iu}{2Q} \int_{-\infty}^{\infty} \frac{e}{\eta - e} de \right)
\]

\[
= \exp \left( -\frac{u \text{sgn}(\eta) \eta}{2Q(u)} + i \left( \frac{u \text{sgn}(\eta) \mathcal{H}(\eta)}{2Q(u)} \right) \right)
\]

The $\mathcal{H}$ denotes the Hilbert transform. The action of attenuation is thus modeled as a pseudodifferential operator. The source wavelet $w$ is first modified by attenuation which depends on the time, and then convolved with reflectivity. The $w_\alpha$ is similar to the Fourier transform of $w$, except for the factor $\alpha(u, \eta)$ in front which describes the attenuation. In $\alpha(u, \eta)$ the negative exponential, $\exp \left( -\eta u / (2Q) \right)$, causes the decay in amplitude and the remaining part represents the phase.

To simplify this to something manageable, perform the following steps. First expand $\alpha(u, \eta)$ into a linear Taylor polynomial approximation. This is valid for small attenuation, or large $Q$. Then integrate only with respect to $\eta$ in the expression for $w_\alpha$. Finally, find $s$, then $s \ast \hat{s}$, and then the Fourier transform $\mathcal{F}[s(t) \ast \hat{s}(t)]$. (This idea is due to Tobias Schaefer).
The Taylor expansion for $\alpha(u, \eta)$ is:

$$\alpha(u, \eta) = \exp \left( \left( \frac{u}{2Q(u)} \right) (-\text{sgn}(\eta)\eta + i \text{sgn}(\eta)\mathcal{H}(\eta)) \right)$$

$$= \sum_{k=0}^{\infty} \frac{1}{k!} \left( \frac{u}{2Q(u)} \right)^k (-\text{sgn}(\eta)\eta + i \text{sgn}(\eta)\mathcal{H}(\eta))^k$$

$$= \sum_{k=0}^{\infty} \frac{1}{k!} \left( \frac{u}{2Q(u)} \right)^k \kappa(\eta)^k$$

where

$$\kappa(\eta) = -\text{sgn}(\eta)\eta + i \text{sgn}(\eta)\mathcal{H}(\eta).$$

In expression for $w_\alpha$, $u$ and $\eta$ are uncoupled so that the integration can be performed only with respect to $\eta$:

$$w_\alpha(u, v) = \sum_{k=0}^{\infty} \frac{1}{k!} \left( \frac{u}{2Q(u)} \right)^k \int \tilde{\omega}(\eta) \kappa(\eta)^k e^{i\eta v} d\eta$$

$$= \int \tilde{\omega}(\eta) e^{i\eta v} d\eta + \left( \frac{u}{2Q(u)} \right) \int \tilde{\omega}(\eta) \kappa(\eta) e^{i\eta v} d\eta + \cdots$$

$$= w(v) + \frac{u}{2Q(u)} J_1(v) + \frac{1}{2} \left( \frac{u}{2Q(u)} \right)^2 J_2(v) + \cdots$$

where

$$J_n(v) = \int \tilde{\omega}(\eta) k^n(\eta) e^{i\eta v} d\eta.$$ 

Now express $s(t)$ in terms of $w$, $r$, $J_1$ and $t$. Using

$$s(t) = \int w_\alpha(\tau, t - \tau) r(\tau) d\tau$$

and

$$w_\alpha(\tau, t - \tau) = w(t - \tau) + \frac{r}{2Q(\tau)} J_1(t - \tau),$$
we have

\[
s(t) = \int w(t - \tau)r(\tau) \, d\tau + \int \frac{\tau}{2Q(\tau)} J_1(t - \tau) r(\tau) \, d\tau
\]

\[
= w \ast r + \int \left( \frac{\tau r(\tau)}{2Q(\tau)} \right) J_1(t - \tau) \, d\tau
\]

\[
= w(t) \ast r(t) + \left( \frac{tr(t)}{2Q(t)} \right) \ast J_1(t).
\]

Similarly,

\[
\tilde{s}(t) = \tilde{w} \ast \tilde{r} + \left( \frac{(tr)_{\text{tilde}}}{2Q} \right) \ast \tilde{J}_1(t).
\]

We assume that \( Q \) is symmetric about \( t = 0 \) s.

We have already seen in the discussion of the white-reflectivity assumption that \( r \) has the nice property that \( r(t) * \tilde{r}(t) = \delta(t) \). To generalize this, use

\[
t^n r * \tilde{r} = \beta_n \delta(t).
\]

Also note that

\[(a \ast b)_{\text{tilde}} = \tilde{a} \ast \tilde{b}\]

and

\[
\mathcal{F}_t \left[ \tilde{s}(t) \right] = \mathcal{F}_t [\delta(t)] = 1 \implies \tilde{s}(t) = \delta(t).
\]

These properties allow us to simplify \( s(t) * \tilde{s}(t) \) as follows:

\[
s(t) * \tilde{s}(t) = w \ast r * \tilde{w} \ast \tilde{r} + \left( \frac{tr}{2Q} \right) \ast J_1 * \tilde{w} \ast \tilde{r} + \left( \frac{(tr)_{\text{tilde}}}{2Q} \right) \ast \tilde{J}_1 * w \ast r
\]

\[
= w \ast \tilde{w} + \left( \frac{tr * \tilde{r}}{2Q} \right) \ast J_1 * \tilde{w} + \left( \frac{(tr)_{\text{tilde}} * r}{2Q} \right) \ast \tilde{J}_1 * w
\]

\[
= w \ast \tilde{w} + \left( \frac{\beta_1}{2Q} \right) \ast J_1 * \tilde{w} + \left( \frac{\beta_1}{2Q} \right) \ast \tilde{J}_1 * w.
\]

The last step is to simplify the Fourier transform of the last two terms in the expression above.

---

1 Observation by Vinicius Anghel: \((a \ast b)_{\text{tilde}} = \tilde{a} \ast \tilde{b}\), although \((ab)_{\text{tilde}} \neq \tilde{a} \tilde{b}\). This corrected one sign in the expression for \( s \ast \tilde{s} \).
Given that
\[ \kappa(\eta) = -\text{sgn}(\eta)\eta + i \text{sgn}(\eta)H(\eta), \]
\[ J_n(v) = \int \tilde{\omega}(\eta)\kappa^n(\eta)e^{iv\eta} d\eta. \]

Calculate
\[ F\left[ \frac{\beta_1}{2Q} J_1 \ast \tilde{w} + \frac{\beta_1}{2Q} w \ast \tilde{J} \right]. \]

First term:
\[ F [J_1 \ast \tilde{w}] = \tilde{J}_1 \tilde{w} = \tilde{w}(\eta)\kappa(\eta)(\tilde{w}^*(\eta)) = \lvert \tilde{w} \rvert^2 \kappa. \]

Second Term:
\[ F [\tilde{w} \ast J_1] = \tilde{w}_1 \tilde{J} = \tilde{J}_1 \tilde{w} = \tilde{w}^* \kappa^* \tilde{w} = \lvert \tilde{w} \rvert^2 \kappa^*. \]

Add them together to obtain
\[ F \left[ \frac{\beta_1}{2Q} J_1 \ast \tilde{w} + \frac{\beta_1}{2Q} w \ast \tilde{J} \right] = \frac{\beta_1}{2Q} \lvert \tilde{w} \rvert^2 (\kappa + \kappa^*) \]
\[ = \frac{\beta_1}{2Q} \lvert \tilde{w}(\omega) \rvert^2 (-\text{sgn}(\omega)\omega). \]

We conclude that
\[ F[s(t) \ast \tilde{s}(t)] = \lvert \tilde{w} \rvert^2 + \frac{\beta_1}{Q} \lvert \tilde{w} \rvert^2 (\text{Re}(\kappa)), \]
\[ (12) \quad \lvert \tilde{s} \rvert^2 = \lvert \tilde{w} \rvert^2 - \frac{\beta_1}{Q} \lvert \tilde{w} \rvert^2 (\text{sgn}(\omega)\omega). \]

This makes sense because then \( \lvert \tilde{s} \rvert^2 \) is real. Also, the amplitude decreases as \( Q \) decreases. If \( Q \to \infty \), then the equation reduces to the classical equation (9).

4.3 A discrete model

It is useful to think about the extent of the ill-posedness in the problem by viewing a slight restatement. In practice, seismic trace data is sampled at discrete time intervals, for a finite duration of time. We therefore describe a discrete version of the convolutional model above: suppose we know the initial source signal, as well as the attenuation and reflectivity properties of the medium being sampled.
Let the data be sampled at times $t_1, t_2, \ldots, t_n$. From this, we can construct a matrix $W_\alpha$, and a vector of reflectivities $\vec{r} = (r_1, r_2, \ldots, r_n)^T$, where $r_i$ is the reflectivity of the layer at depth $ct_i$. Then, the discrete version of equation (11) is

$$W_\alpha \vec{r} = (\vec{w}_1 \mid \vec{w}_2 \mid \ldots \mid \vec{w}_n) \vec{r} = \vec{s} := (s_1, s_2, \ldots, s_n)^T.$$ 

The entries $w_{ij}$ of matrix $W_\alpha$ have the following properties:

- If $t_i > t_j$, $w_{ji} = 0$ (causality assumption).
- If $t_i < t_j$, $w_{ji} = w_\alpha(t_i, t_j - t_i)$ where $w_\alpha$ was defined in equation (11).

Therefore, $W_\alpha$ is lower triangular, and the amplitude spectra of column vectors $\vec{w}_i$ attenuate by an exponential factor from left to right.

The forward seismic problem is: given $W_\alpha$, $\vec{r}$, find the seismic trace vector $\vec{s}$.

The inverse seismic problem is: given $\vec{s}$, find $W_\alpha$, $\vec{r}$. In our specific case, we have to find $W_\alpha$, specifically the amount of attenuation between the amplitude spectra of the columns of $W_\alpha$. As is easy to see, the inverse problem is quite ill-posed.

4.4 Experiments with the convolutional model of attenuation

A few coding experiments were done using the Wiener model with attenuation.

From equation (9), in the case without attenuation we see that in the frequency domain, the amplitude of the collected signal is the same as the amplitude of the source signal. The first step was to see how the amplitude of the collected signal compares with $|\hat{w}(\omega)|$ if attenuation is present. For the synthetic trace, the $w$ is known, so we can calculate $\mathcal{F}[s \ast \tilde{s}]$ to see how it compares with $|\hat{w}(\omega)|$ for different $Q$’s. The $\mathcal{F}[s \ast \tilde{s}]$ and $|\hat{w}|^2$ values were normalized by dividing by their respective mean values. The relative error is calculated as the absolute difference divided by the maximum of $|\hat{w}|^2$. The difference between the two is very big and decreases only slightly as $Q$ increases (Figure 9). Note that as expected from equation (9), the spectrum of the signal in the frequency domain has the same basic shape as the spectrum of the source wave, with superimposed rapid, small amplitude oscillations.

There are two ways to calculate $\mathcal{F}[s \ast \tilde{s}]$: either by taking a Fourier transform (in our case, Gabor transform) of a convolution, or by calculating only the the Fourier (Gabor) transform of $s$ and multiplying it by the conjugate. From now on, $\mathcal{G}$ will be used to denote a Gabor transform. To test that $\mathcal{G}[s \ast \tilde{s}]$ was calculated correctly, we calculated it
using the two methods. For the method using the convolution, however, the result oscillates from negative to positive at every point, so that multiplying it by a vector of \([1, -1, 1, -1, \ldots]\) gives a positive result. This means that there is a mistake in the phase. After this correction, the two methods give the same result. So from now on \(G'[s]G[s]\) is calculated as \(G'[s]G[s]\), where the superscript \(^*\) denotes conjugation and \(*\) denotes convolution.

The next step was to see how \(G'[s]G[s]\) changes over time for the normal and abnormal trace. First, the time is divided into overlapping windows, at which \(G'[s]G[s]\) is calculated. For each time window, store the maximum value of \(G'[s]G[s]\). Taking the maximum might not be the best method because \(|G'[s]|\) has its maximum at low frequency, and we are interested in how the high frequencies decay. But we do see that for the abnormal trace, the value of \(G'[s]G[s]\) is lower than that for the normal trace after \(t = 1\ s\) where the abnormality occurs (Figure 10). This is consistent with equation (12), because since \(Q\) is smaller at the abnormality, \(1/Q\) is larger, and the value of \(G'[s]G[s]\) decreases.

When this process is applied to the real trace, the resulting graphs seem similar to those for \(R_j b_s g_f(\omega, t_j)\), recorded in the process of using the centroid method. (Figure 11, compared to Figure 4(a) and Figure 5(a). They are also similar to the wavelets and polynomial fit graphs.

To see how \(G'[s]G[s]\) varies as \(Q\) varies, its maximum in one window was calculated for different values of \(Q\) and plotted with respect to \(1/Q\). We would like the relationship to be linear, though we expect it not to be since higher order terms are ignored (Figure 12).

After these preliminary investigations, we would like to get back to equation (12) to find a way to use it for estimating \(|\hat{w}|^2\) and then \(Q\). If we use the Gabor transforms, for our discrete case we have that

\[
G[s * \hat{s}] = |\hat{w}(\omega, t_i)|^2 - \frac{\beta_1}{Q(t_i)} |\hat{w}(\omega, t_i)|^2 \text{ (sgn}(\omega)\omega)
\]

where \(\omega\) is the frequency. We can use \(G'[s]G[s]\) from a window near the beginning of the trace to estimate \(|\hat{w}|^2\), since the signal has not traveled a far enough distance to be attenuated greatly. Dividing the \(G'[s]G[s]\) value from every other window by the estimate of \(|\hat{w}|^2\) obtained from the first window should give an estimate of \(1 - \frac{\beta_1}{Q(t_i)} \text{ (sgn}(\omega)\omega)\). We are assuming that \(|\hat{w}|^2\) changes little in time, and that the value of \(G'[s]G[s]\) is affected most by changes in \(Q\). The values obtained for the abnormal trace are smaller than those for the normal trace (Figure 13). There are large spikes due to dividing by small numbers.
An idea for future work would be to find an approximation to $|\widehat{w}|^2$ by taking a Fourier transform of the entire trace. Then this value can be compared to the $G^*[s]G[s]$ of each window along the trace. Clearly the
method needs to be refined before tests on the real data are meaningful.

5 Wavelets applied to the seismic trace Multi-resolution analysis can be used to remove noise from a seismic trace. Random noise and the reflectivity response of the earth are both high frequency components of the signal. The low-frequency approximation to the signal filters out the noise and reflectivity, leaving only the information on the attenuation and the source function. De-noising using wavelets amplifies the difference between signals that have suffered different amounts of attenuation [15]. Unfortunately, the effect of attenuation is more pronounced in the high-frequency end of the spectrum, which is filtered out during denoising.

The high frequency components of a signal can also be treated in a similar manner as low frequency components. Since the attenuation dampens high frequencies, abrupt changes in the detail components may indicate changes in attenuation. For the synthetic data, the second and third detail component reconstructions of the anomalous trace have smaller amplitude after time $t = 1$ s (Figure 14). The discrete Meyer wavelet was chosen by trial and error, because the amplitude difference between the anomalous and normal traces seemed largest.
(a) Pike Peaks reservoir

(b) Blackfoot reservoir

FIGURE 11: The maximum value of $G^*[s]G[s]$ over many different windows for the real data.
The fine detail reconstruction for the Pike Peaks data shows greatest variation at $t = 0.65$ s, traces 300 to 500 (Figure 15). This is similar to the location found from the graphs of frequency shift and polynomial fit. This location is slightly above the location of the reservoir. The second and third detail reconstructions (not shown) are similar to the first detail reconstruction, though the irregularity is more clearly featured. The fourth detail reconstruction shows another area of variation closer to the position of the reservoir. A region of large oscillation appears below the $t = 0.6$ s region, at $t = 0.7$ s and around trace 400.

For the Blackfoot data there are faint irregularities in the oscillations. The fourth detail reconstruction contains horizontal regions of greater variation at $t = 1.1$ s, traces 55–75 and at $t = 1.7$, traces 10–30. The area at $t = 1.1$ s is close to the anomaly in the Blackfoot cubic coefficient graph.

6 Denoising using a minimization technique  

Recall that we are seeking to locate $-A(\omega, t)$ from noisy data $d$, where

$$d = \log |\tilde{s}| = -A(\omega, t) + \log(|\tilde{f}|) = m + n.$$ 

We have already discussed a wavelet-based denoising strategy above. Another possible means of removing the noise from the data is to solve
FIGURE 13: An estimate of $|\hat{\omega}|^2$ was obtained from the Gabor transform from the first window. The Gabor transform of every other window was divided by this value, in order to give an estimate of $1 - \frac{1}{Q(t)} (\text{sgn} (\omega) \omega)$ for that time. These windows are at $t = 1.08$, at the time corresponding to the low $Q$ region for the abnormal trace.
FIGURE 14: The multi-resolution decomposition of the synthetic traces. The abnormal trace has smaller amplitudes in the detail graphs after $t = 1 \text{s}$. 
FIGURE 15: Some of the detail reconstruction using wavelets for the Pike Peak and Blackfoot data.
FIGURE 15: (contd.) Some of the detail reconstruction using wavelets for the Pike Peak and Blackfoot data.
a minimization problem

\[
\min_m \frac{1}{2} \| C_n^{-1/2}(d - m) \|^2_2
\]

where \( C_n \) is the covariance matrix of the noise \( C_n = E[nn^T] \). We can use a wavelet transform \( W \) to convert the minimization problem into

\[
\min_{\tilde{m}} \frac{1}{2} \| \Gamma^{-1}(\tilde{d} - \tilde{m}) \|^2_2 + \lambda \| \tilde{m} \|_p
\]

where

\[
\tilde{d} := Wd, \quad \tilde{m} := Wm, \quad \tilde{n} := W(n), \quad E(\tilde{n}\tilde{n}^T) = \Gamma^2.
\]

The noise thresholding used is called hard or soft, depending on whether \( p = 1 \) or \( 2 \), respectively. Solving the minimization problem and subsequently inverting would allow us to reconstruct an approximation to \( m \).

This strategy has not yet proved computationally efficient, and remains only a possible future direction.

7 Conclusions and future work This paper explores different signal analysis techniques for reservoir location: the centroid method, polynomial fit, wavelet analysis techniques, and the modified Wiener method.

Many of the results from different techniques were consistent. For the Pike Peaks data, we observe two regions of irregularity. The one above the location of the reservoir at \( t = 0.6 \) s appears in the centroid method, the polynomial fit, and the wavelet method. The lower area, at the location of the reservoir, is sometimes more difficult to find. The contrast scale for the centroid method, the order of the polynomial, or the level of detail for the wavelet method have to be adjusted correctly. The Blackfoot data is even more sensitive to these adjustments.

The Wiener technique is promising because, like for the centroid method, the theory of the method is directly used in its application; it is less heuristic than the wavelet method or the polynomial fit. We developed a theoretical model for the Wiener method in the presence of attenuation. We arrived at the equation

\[
|\hat{s}|^2 = |\hat{w}|^2 - \frac{\beta_1}{Q} |\hat{w}|^2 \text{sgn}(\omega) \omega
\]
which can be used to determine how $Q$ changes over time. However, most of the work done so far was directed at understanding the background for the method (e.g., the general behaviour in time of $G^*[s]G[s]$). The next step would be to implement the estimation of $|\tilde{\omega}|^2$ from the Fourier transform for the entire trace and using this to find how $\frac{\partial}{\partial t} (\text{sgn}(\omega)\omega)$ changes. Also, the method should be tested on the real data.

Another promising direction is the use of change-point methods from statistics to analyze these problems, which is the subject of ongoing work.

REFERENCES


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