The ABC's of Graduate Studies in Mathematics

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Abstract: The transition from Undergraduate studies to Graduate school in Mathematics is marked through various changes related to learning, teaching, research, and the organization of student's lives. In order to be successful in your program, it is important to know what is expected and how to best manage the new challenges of a graduate program. This article shall help students to get ready for graduate school in Mathematics. My experiences are based on the graduate program of the Department of Mathematical and Statistical Sciences at the University of Alberta. My recommendations, however, are valid for most North-American universities and beyond.

1 Introduction

Every year hundreds of students enter an MSc or PhD program in Mathematics in a North-American university. During their Undergraduate program, they were exposed to intense training in Mathematics, Statistics and other areas. The teaching in undergraduate schools, however, is quite different from graduate school and students are up for a change. In Undergraduate school the typical Mathematics class is taught on the blackboard and the students are constantly tested through quizzes, homework assignments and exams. Students can be lucky if they encounter alternative teaching methods, for example classes where the grades are based on course work, on oral presentations, or, even better, on research related activities. There are many such initiatives throughout the University system and they help to prepare students for research and for graduate school. Typically, beginning graduate students know quite well how to learn and how to prepare for exams. They are, however, not so experienced in other aspects of graduate school, such as independent research, critical thinking, creativity, presentation, and writing skills.

2 Courses, Exams, and Scholarships

The goal of a graduate program in Mathematics is to lead students into active research. Since students already have a BSc (or equivalent) degree and they do not necessarily need another degree to find a job students are typically self-motivated. Hence the need for rigorous testing fades out. Of course, there are still scholarships which are based on the student's GPA and there are still exams, but this is of lesser and lesser importance. In graduate school you will have courses which have no exams at all. Sometimes a term paper is needed or a presentation at the end of term. You will also encounter high level graduate courses where

the grades range from A- to A+. It is no longer assumed that the student community is normally distributed with a mean at a B. If all students are good, they all get a good grade. Often you get homework problems which are not checked. The instructor assumes that you do them since you want to learn the material and not since you need a good grade.

Many schools have entrance year requirements, entrance exams, candidacy exams, qualifying exams etc, etc. Some schools feel that these exams are necessary, since students come from all over the world (China, USA, Canada, Europe, etc.) and they have very different backgrounds. These exams need to be taken very seriously as they are used to filter out weak students. You might find it helpful to talk to more experienced students and/or to try to find earlier versions of the exams to practice with.

The whole issue of scholarships is an interesting topic and there are many different forms of scholarships; some are major and provide funding for the full year, others are partial and help with tuition or travel expenses. It is worthwhile to inform yourself about scholarships and scholarship deadlines. Many Universities have websites which are dedicated to scholarships. Scholarships are very important for the students. They provide financial independence and allow more freedom to focus on research work. However, the evaluation process for scholarships demands rankings, GPA's, and course grades. In that respect, scholarships are undermining the idea of a free graduate school as outlined above. Fortunately, more and more scholarships do not only look at the GPA but other aspects, such as publications, presentations, innovation, citizenship, and other activities are taken into consideration. But then, a good GPA is always a necessity for a successful scholarship application.

3 The three C's of Graduate Studies

The goal of a graduate program in Mathematics can be summarized as the three **C**'s: Students should learn to be **competent**, **creative**, and **critical**.

Be Competent: It is, of course, a major goal of the graduate program to teach the students new and interesting material. At the end of your degree you are expected to be an expert in something, be it algebra, groups, probability, differential equations, fluids, mathematical biology etc. During your whole educational path you have learned how to learn, hence I do not need to discuss learning of new and complicated material in this article.
Competence also includes a responsibility for your own learning. First you need to understand the graduate program which you signed up for. It is your responsibility to know which courses you have to take and which are optional. You should also know about deadlines, exams, special needs arrangements for disabilities, special sport events, or pregnancies and parental leaves, whatever applies to you. And, of course, you are responsible to follow the ethical standards of your school.
In graduate school, learning is no longer restricted to the immediate course material. Often you have to be resourceful and go way beyond what is expected. For example if you don't remember what "compact" means, then you need to look it up; if the difference between "homomorphism" and "homeomorphism" is not clear, then you do not need to bother an instructor - you can find it yourself. If you don't remember if the normal distribution has a factor of $\frac{1}{\sigma\sqrt{2\pi}}$ or $\frac{1}{\sqrt{\sigma 2\pi}}$ then look it up and try to memorize. When I was a student, I had the most important formulas, for example the Laplacian in polar coordinates, written on a

piece of paper and pinned to the wall. (not sure what "Laplacian" is? go and look it up!).

It can be confusing for students to do proofs in homework or exams. Most graduate students come from honors programs and they have learned how to argue a proof. However, other students come from other areas (general science, physics, biology for example) and in their courses the detailed study of proofs has been abandoned. I believe that **being able to formulate a proof is the signature skill of a mathematician** and it is of outmost importance to acquire this skill!

For example a proof "A implies B" cannot start with "Assume B...". Another example: What is the negation of

$$\forall\, \varepsilon > 0\ \exists\, \delta > 0 \text{ such that } |f(x) - f(y)| < \epsilon\ \forall\, |x - y| < \delta. \tag{1}$$

Then the negation is

$$\exists\, \varepsilon > 0 \text{ such that } \forall\, \delta > 0\ \exists |x - y| \leq \delta \text{ such that } |f(x) - f(y)| \geq \varepsilon. \tag{2}$$

Try to find such a function!

Be Creative: Here comes something new – at least for many students. To do research, you have to go beyond pure knowledge. You have to learn to apply your knowledge to open problems. I have to admit that for some MSc programs a real contribution to research is not needed. But let's talk about programs, which do include a research component. Most thesis based MSc programs and all PhD programs require research. Beyond knowledge of your discipline, you need to be able to draw connections between apparently unrelated topics; you need to take ideas from this area to the other area; you need to be able to develop new ideas.

Come up with your own definitions and see how far they carry. In my MSc thesis, I came up with the definition of "potty" (which has nothing to do with toddler training). The definition was useful for my thesis, as it allowed me to formulate my results, however, it was certainly not of any importance beyond my thesis, but that's fine.

Creativity is very important for a PhD thesis. I have never seen a PhD thesis which did not include several original ideas. In most cases the supervisor expects that the student comes up with his or her own ideas - ideas the supervisor did not have. Here lies a major difference between undergraduate and graduate education. A PhD student is expected to generate ideas!!

Be Critical: Textbooks from undergraduate courses are often in their sixth, seventh, eight edition and you can be certain that all mistakes have been found and corrected. This is not necessarily the case for graduate material. Some course notes are fresh from the faculty member, just written for this course. Some course notes are adapted from recent publications, and of course, it is possible that they contain mistakes. It is very hard for a student to admit that there indeed is a typo, or a wrong conclusion in a paper. After a student has checked a questionable conclusion many times over then he or she should consider the possibility that indeed it is wrong. At this point it should be discussed with the supervisor.

Being critical is more than finding mistakes. Students should also challenge ideas. Ideas from other students, ideas from papers and ideas from their supervisor. I claim, for example, that there is no continuous function which satisfies the above statement (2). You can assume that I have good reasons for my claim, but you should not readily believe me. Challenge my claim until you are either convinced or until you disproved me.

A word to Chinese students: In my experience, Chinese students are very well prepared for graduate studies and they are typically among the best students. However, they seem to have particular problems in critical thinking and creativity. During a graduate program they need to learn that not everything the supervisor says is correct and they need to learn to trust their own ideas. This is not easy and you need to work on yourself.

4 Some Survival Tips

Here I describe some basic rules to be successful in your graduate program.

Exams: One important aspect is course work and exams. You should prepare for exams similarly to the way you did for your undergraduate courses. Since you were admitted to a gradate program, you have already shown that you are capable of learning and writing exams.

Supervisor: Equally important is your relation to your supervisor. Be prepared for meetings with the supervisor and make a list of thinks to do during the meeting. These include question about a paper you are reading, trials and errors on a proof you are working on, discussion of new ideas which you came up with last week, advising, suggestions etc. Come to your supervisor with questions, suggestions, and ideas. Never ask you supervisor **"What shall I do next?" - this question is forbidden!!!!** Instead ask like this: "I did this and this ... and here is an argument I did not understand and I would like to read more about it" - or - "this aspect was quite interesting and I was wondering if the next step could be to do ..." - or - "the claim of Dr. Soandso can't be right, I think we should find a counter example ...". Your supervisor will appreciate your initiative and he or she will encourage you to follow a lead and he or she will stop you if he or she thinks your idea goes nowhere.

I have to admit that I do have a colleague who gives PhD students well defined assignments. Although I don't approve of it, there is not much I can do about it. There are always exceptions.

Papers: An important part of research is reading original papers. In 2003 I published the following guideline in "Pi in the Sky":

"How to read a mathematical paper?

Once I asked a student how she reads a mathematical paper and she said: "I sit at my desk and stare at it for a very long time - eventually I will understand - hopefully."
Well, there are certainly many ways to read a mathematical paper. The method which works pretty good for me is the following:

1. Read the paper straight through. Don't bother about the mathematical details. Try to

understand what's this is about, what is the result, what is the point?

2. Now check the details. Take some blank paper and a pencil and follow all the calculations and modifications. This is the only way to get a deep understanding of the paper!

3. As all details are checked, read it again. What methods are used? What is the basic idea of the proof?

4. And if you like you can ask further, like: Can it be generalized? Can the method or result be applied to some other problem? Can I shorten the proof? Would a different method be more (or less) efficient ? etc. And ultimately you start your own research..."

5 Presentation and Writing Skills

There is a lot of literature on academic writing and presentation skills and I would be unable to summarize all ideas and suggestions which are discussed. What I can add to these discussions are four simple rules, which are based on my own personal experiences:

- Presentation skills can be learned and you need to practice. Try to choose classes which have a presentation component and volunteer for presentations in reading groups, seminars, and journal clubs.

- Often students wonder how to write a thesis. The advice is easy. Write at a level of detail which you are comfortable with. Do not try to sound overly complicated, rather try to be as clear as possible. If it's not clear to you it is certainly not clear to others.

- Take advantage of writing or presentation classes which are offered in many schools.

- Improve your English skills.

6 Get Help!

The challenges of graduate school are often combined with other issues which can make life complicated: financial commitments, tuition, rent, a new environment, a new culture, change of friendships and relationships, personal handicaps and illnesses. For all these aspects of student live there are institutions and organizations which help you. Do not hesitate to ask for advice and help!

7 Summary

I hope that this report can guide incoming graduate students to be successful in Mathematics. I am sure many of the topics mentioned above are relevant to other disciplines as well, for example Statistics. I am, however, not expert enough in other areas to extend my recommendations beyond mathematics.

During your graduate program, remember the three C's:

be competent, be critical, be creative!

... and an "F":

have fun!