Recommended Background Material for First-Term Graduate Courses

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Introduction

The Department of Mathematical and Statistical Sciences at the University of Alberta is one of the strongest research-oriented mathematics departments in Canada. Our faculty include leading experts in various areas of mathematics and statistics. Each year we attract excellent graduate students who contribute to our research efforts. Our graduate students come from all over the world and have very different background knowledge. Many come from Honors programs in Mathematics, but we also admit students from Biology, Engineering, Physics, Computer Science and other areas. We are proud of attracting this wide, interdisciplinary mix of people and talents.

Students are often surprised by the high level of mathematical rigor expected of them in all of our graduate courses. Keep in mind that being able to prove something rigorously is a signature skill of a mathematician. Don't be afraid of proofs but rather learn the skill to read, understand and design them on your own. To be successful in mathematics it is neither required nor sufficient to be a good programmer.

In Statistics, we offer an entrance exam for the PhD program, to see what background material you might be missing and to guide you in your choice of courses. In Mathematics, we do not have such an entrance exam and we do not intend to introduce one. Instead, we offer this guideline to the necessary background material. Should you have further questions please feel free to contact our Graduate Coordinator or the instructors of specific courses. It is your responsibility to make sure you have the required background knowledge for all the courses you are taking.

Our first year graduate courses serve three purposes. Firstly, they introduce to a given subject area. Secondly, they are designed to bring everybody onto the same level of proficiency. Inevitably, some students find this process rather easy, while others find it quite challenging. Thirdly, your performance in these courses is being monitored and used to evaluate your progress. To be successful, we strongly recommend that you make yourself familiar with the background material listed below.

The references given are subjective choices. For each and every reference there are many alternatives of comparable quality, and it is impossible to list them all. For example, to bolster up your background in real analysis, you do not have to use Rudin's book [Ru1] — many other texts on the subject will do just as well. Also, please bear in mind that you are by no means supposed to memorize any of the textbooks given as references. You should merely feel comfortable with the material covered.

When looking through the background material for the courses listed below, you will find three recurring core themes, namely *Real Analysis*, *Linear Algebra*, and *Algebra*. A sound knowledge in these subjects is indispensable for all graduate studies in mathematics. Accordingly, we recommend the following three excellent, time-tested textbooks to every budding graduate student:

[Ru1] W. Rudin, *Principles of Mathematical Analysis*, 3rd ed, McGraw-Hill, 1976.

[Ax] S.J. Axler, *Linear Algebra Done Right*, Springer, 1997 (available online via the U of A Library).

[La2] S. Lang, *Algebra*, Springer, 2002.

In addition to these, you will find a number of more specialized references, for example on differential equations, number theory, measure theory, etc.

Remember that this guideline is not meant to put you off. Rather, it offers you the chance of coming to your first class well prepared, and thus of taking an important first step towards a successful and enjoyable time in our MSc and PhD programs.

Edmonton, March 2010
T. Hillen, A. Berger

Math 506 – Complex Variables IIA

Required background

- An undergraduate course in complex analysis;
- Real analysis;
- Basic notions in algebraic topology (homotopy, fundamental groups, homology);
- Basic notions in algebra (groups, rings, modules, fields);
- Basic notions in differential geometry (differential forms, Stokes' theorem).

Recommended reading

[BT] R. Bott and L.W. Tu, *Differential Forms in Algebraic Topology*, 3rd ed, Springer, 1995.

[La2] S. Lang, *Algebra*, Springer, 2002.

[Roy] H.L. Royden, *Real Analysis*, 3rd ed, Prentice-Hall, 1988.

[Ru2] W. Rudin, *Real and Complex Analysis*, 3rd ed, McGraw-Hill, 1986.

Math 512 – Algebraic Number Theory

Required background

Acquaintance with groups and rings, including in particular:

- Basic linear algebra;
- The structure of finitely generated abelian groups;
- Basic principal ideal domains;
- Finite extensions of fields;
- The rudiments of Galois theory up to the Galois correspondence;
- a little basic analysis.

Recommended reading

[Ar] M. Artin, *Algebra*, Prentice-Hall, 1991.

[DF] D. Dummit and R. Foote, *Abstract Algebra*, 2nd ed, Wiley, 2008.

[J] N. Jacobson, *Basic Algebra 1*, 2nd ed, W.H. Freeman, 1985.

[La2] S. Lang, *Algebra*, Springer, 2002.

Math 515 – Mathematical Finance I

Required background

- Measure and Integration;
- Elementary Probability Theory;
- Mathematical Foundation to Probability Theory;
- Convergence of Probability Measures: Central Limit Theorem.

Recommended reading

[N] J. Neveu, *Mathematical Foundation of the Calculus of Probability*, 1965: Chapters 1–4.

[Sh] A.N. Shiryaev, *Probability*, Springer 1996: Chapter 1, Chapter 2 (Sections 2–8), and Chapter 3 (Sections 1,3,4 and 5).

Math 516 – Linear Analysis

Required background

- Linear algebra;
- Elements of set theory
- Real Analysis (including Lebesgue measure and integral);
- Introduction to complex analysis;
- Introduction to topology, metric spaces, topology on metric spaces.

Recommended reading

[Roy] H.L. Royden, *Real Analysis*, 3rd ed, Prentice-Hall, 1988.

[Ru2] W. Rudin, *Real and Complex Analysis*, 3rd ed, McGraw-Hill, 1986.

[SS2] E.M. Stein and R. Shakarchi, *Real Analysis, Measure Theory, Integration, and Hilbert Spaces*, Princeton Lectures in Analysis, III. Princeton University Press, 2005.

Math 521 – Differential Manifolds

Required background

- A good knowledge of advanced calculus of several variables, linear algebra and some modern algebra;
- A basic understanding of point set topology;
- A course in Differential Geometry of Curves and Surfaces.

Topics covered

Topologies, bases, subspaces, product spaces, quotients, group actions, compactness, connectedness. The differential of a map, Inverse Function Theorem, Implicit Function Theorem, Rank Theorem, derivatives of higher order. Parametrized (regular) curves and surfaces, local theory of curves and surfaces, local theory of submanifolds in $\mathbf{R}^n$.

Recommended reading

[dC] Manfredo P. do Carmo, *Differential Geometry of Curves and Surfaces*, Prentice-Hall, 1976.

[Da] R.W.R. Darling, *Differential Forms and Connections*, Cambridge University Press, 1999: Sections 1–4.

[E] C.H. Edwards Jr., *Advanced Calculus of Several Variables*, Dover, 1994.

[Le] J.M. Lee, *Introduction to Topological Manifolds*, Springer, 2000: Sections 1–4.

[Ru1] W. Rudin, *Principles of Mathematical Analysis*, 3rd ed, McGraw-Hill, 1976: Sections 9 & 10.

Math 524 – Ordinary Differential Equations

Required background

- Real analysis on $\mathbf{R}^d$; norms, continuity, differentiation, integration, uniform convergence;
- Mean Value and Implicit Function Theorems;
- Basic linear algebra; linear spaces and maps, bases, matrices, linear equations, determinants, eigenvalues and eigenvectors, diagonalisation, inner products;
- Abstract analysis on metric spaces; convergence, continuity, completeness, compactness, connectedness;
- Contraction Mapping Theorem;
- Basic differential equations; Separable and linear equations, elementary integration techniques, solutions, phase portraits for 1-D problems.

Reference for Background Material

[Ax] S.J. Axler, *Linear Algebra Done Right*, Springer, 1997 (available online via the U of A Library).

[BDP] W.E. Boyce and R.C. DiPrima, *Elementary Differential Equations and Boundary Value Problems*, 8th ed, Wiley, 2005.

[Ru1] W. Rudin, *Principles of Mathematical Analysis*, 3rd ed, McGraw-Hill, 1976.

[SV] S. Shirali and H.L. Vasudeva, *Metric spaces*, Springer, 2006 (available online via the U of A Library).

[St] S.H. Strogatz, *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering*, Addison-Wesley, 1994.

Math 527 – Intermediate Partial Differential Equations

Required background

- Multivariable Calculus, in particular: Gauss' Theorem;
- Basic Real Analysis, in particular: Almost everywhere convergence; Fatou, monotone, dominated convergence theorems; Basic L^p space theory;
- Introduction to ordinary differential equations;
- Introduction to partial differential equations;
- Basic Linear Algebra and Functional Analysis; Know the following: Linear vector spaces, Hilbert spaces, Banach spaces.

Recommended reading

[BDP] W.E. Boyce and R.C. DiPrima, *Elementary Differential Equations and Boundary Value Problems*, 8th ed, Wiley, 2005.

[Ha] R. Haberman, *Applied Partial Differential Equations*, 4th ed, Prentice-Hall, 2003.

[Roy] H. Royden *Real Analysis*, 3rd ed, Prentice-Hall, 1998.

[Ru2] W. Rudin, *Real and Complex Analysis*, 3rd ed, McGraw-Hill, 1986.

Math 535 – Numerical Methods I

Required background

- Calculus;
- Linear Algebra;
- Differential Equations;
- Computing and Programming experience.

Recommended reading

[Ax] S.J. Axler, *Linear Algebra Done Right*, Springer, 1997 (available online via the U of A Library).

[BDP] W.E. Boyce and R.C. DiPrima, *Elementary Differential Equations and Boundary Value Problems*, 8th ed, Wiley, 2005.

[Ru1] W. Rudin, *Principles of Mathematical Analysis*, 3rd ed, McGraw-Hill, 1976.

Math 542 – Fourier Analysis

Required background

- Elementary set theory;
- Topology in $\mathbf{R}^d$;
- Lebesgue Measure theory;
- Integration theory;
- Differential theory; the Hardy-Littlewood maximal functions;
- Metric spaces; Banach and Hilbert spaces;
- Properties of L^p spaces.

Recommended reading

[Roy] H.L. Royden, *Real Analysis*, 3rd ed, Prentice-Hall, 1988.

[SS1] E.M. Stein, and R. Shakarchi, *Fourier Analysis. An Introduction.* Princeton Lectures in Analysis, I. Princeton University Press, 2003.

[SS2] E.M. Stein and R. Shakarchi, *Real Analysis, Measure Theory, Integration, and Hilbert Spaces*, Princeton Lectures in Analysis, III. Princeton University Press, 2005.

[WZ] R.L. Wheeden and A. Zygmund, *Measure and Integral. An Introduction to Real Analysis.* Pure and Applied Mathematics, Vol. 43. Marcel Dekker, 1977.

Math 543 – Measure Theory

Required background

- Elements of set theory;
- Real Analysis (including Lebesgue measure and integral).

Recommended reading

[Roy] H.L. Royden, *Real Analysis*, 3rd ed, Prentice-Hall, 1988.

[Ru2] W. Rudin, *Real and Complex Analysis*, 3rd ed, McGraw-Hill, 1986.

[SS2] E.M. Stein and R. Shakarchi, *Real Analysis, Measure Theory, Integration, and Hilbert Spaces*, Princeton Lectures in Analysis, III. Princeton University Press, 2005.

[WZ] R.L. Wheeden and A. Zygmund, *Measure and Integral. An Introduction to Real Analysis.* Pure and Applied Mathematics, Vol. 43, Marcel Dekker, 1977.

Math 556 – Introduction to Fluid Mechanics

Required background

Although this first course in Fluid Mechanics is essentially self-contained, the student is expected to have a basic mathematical training including:

- A basic knowledge of Advanced Multivariable Calculus including Gauss' Theorem and Stokes' Theorem;
- A basic knowledge of Differential Equations including both ordinary and partial differential equations, the method of characteristics, separation of variables and Fourier series;
- Knowledge of Applied Linear Algebra including matrices and their manipulation.
- Some knowledge of Physics is a definite asset.

Recommended reading

[Ax] S.J. Axler, *Linear Algebra Done Right*, Springer, 1997 (available online via the U of A Library).

[Ha] R. Haberman, *Applied Partial Differential Equations*, 4th ed, Prentice-Hall, 2003.

[Ru1] W. Rudin, *Principles of Mathematical Analysis*, 3rd ed, McGraw-Hill, 1976: Sections 9 & 10.

Math 581 – Group Theory

Required background

- Groups (definition, basic properties, examples);
- Homomorphisms, isomorphisms;
- Normal subgroups, quotient groups and isomorphism theorems;
- Linear algebra (in particular, the theory of eigenspaces/eigenvectors of a linear transformation).

Recommended reading

Any introductory algebra textbook will do. The following is a collection of books which of course include much more than the prerequisites listed above (listed in the order of increasing difficulty/depth).

[La2] S. Lang, *Algebra*, Springer, 2002.

[Ar] M. Artin, *Algebra*, Prentice-Hall, 1991.

[Hu] T. Hungerford *Algebra*, Graduate Texts in Mathematics 73, Springer, 1973.

[Rob] D.J.S. Robinson, *A course in the theory of groups*, 2nd ed, Graduate Texts in Mathematics 80, Springer, 1996.

[Ar] also contains most of the linear algebra background needed; so does [Hu], but on a more advanced level.

Math 582 – Rings and Modules

Required background

- Linear algebra;
- Basic notions of rings, fields, ideals and homomorphisms;
- Kernels, images and cokernels of homomorphisms;
- Equivalence relations;
- A good amount of familiarity with the notion of quotient, e.g. the quotient of a vector space by a subspace, of a ring by a (two-sided) ideal;
- The three fundamental isomorphism theorems on quotients and homomorphisms;
- Prime and maximal ideals;
- Polynomial rings (irreducibility, roots, factors, unique factorization, greatest common divisor);
- Finitely generated abelian groups;
- Congruences in the ring of integers and in polynomial rings, the Chinese Remainder Theorem for integers;
- Integral domains (and their quotient fields), Euclidean domains, principal ideal domains, unique factorization domains;
- Tensor product of vector spaces;
- Quadratic extensions of the field of rational numbers.

Recommended reading

[Co] P.M. Cohn, *Classic Algebra*, Wiley, 2000.

[Hu] T. Hungerford *Algebra*, Graduate Texts in Mathematics 73, Springer, 1973.

[Ir] R.S. Irving, *Integers, Polynomials and Rings*, Springer, 2003.

[La1] S. Lang, *Undergraduate Algebra*, Springer 1989.

[Si] L.E. Sigler, *Algebra*, Springer, 1977.

The following books are more advanced, but some parts of them may be useful to prepare for Math 582.

[DF] D. Dummit and R. Foote, *Abstract Algebra*, 2nd ed., Wiley, 2008.

[La2] S. Lang, *Algebra*, Springer, 2002.