Multiscale dynamics and information: from random to data driven dynamical systems

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This talk outlines multiscale problems which combine techniques of model reduction and filtering. Multiple time scales occur in models throughout the sciences and engineering, where the rates of change of different variables differ by orders of magnitude. The basis of this work is a collection of limit theories for stochastic processes which model dynamical systems with multiple time scales.

The *first objective* of this talk is concerned with certain methods of *dimensional reduction* of nonlinear systems with symmetries and small noise. In the presence of a separation of scales, where the noise is asymptotically small, one exploits symmetries to use recent mathematical results concerning *stochastic averaging* to find an appropriate lower-dimensional description of the system. The unique features of the problem are interactions between bifurcations, resonances, dissipation and random perturbations. Bifurcations are where small changes in a system result in large changes in the structure of the fast orbits. The subtleties of the interaction between these effects lead to new and novel analytical techniques. Hence, we are developing techniques of stochastic dimensional reduction to find a simpler model which predicts or captures relevant dynamics of the system. One of the preeminent modern frameworks for considering convergence of the laws of Markov processes is that of the *martingale problem*, which we use in deriving the coarse-grained dynamics.

State estimation of random dynamical systems with noisy observations has been an important problem in many areas of science and engineering. Since the true state is usually hidden and evolves according to its own dynamics, the objective is to get an optimal estimation of the true state via noisy observations. The theory of filtering attempts to give a recursive procedure for estimating an evolving signal or state from a noisy observation process. The *second objective* of this talk is to develop, with mathematical rigor, a lower - dimensional nonlinear filter by combining two ingredients, namely, stochastic dimensional reduction discussed above and nonlinear filtering. We find a reduced nonlinear filtering problem when the system dimension can be reduced via homogenization. We approximate the complex original nonlinear filtering equations by simpler ones with a quantifiable error. This talk is focused on some of the theoretical aspects that deal with reduced dimensional nonlinear filters. In particular, we show how scaling interacts with filtering. Roughly speaking, we show the efficient utilisation of the low-dimensional models of the signal and develop a low-dimensional filtering equation. We achieve this through the framework of homogenisation theory which enables us to average out the effects of the fast variables.

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