NONLINEAR STRUCTURAL STABILITY AND LINEAR DYNAMIC INSTABILITY OF TRANSONIC STEADY-STATES TO A HYDRODYNAMIC MODEL FOR SEMICONDUCTORS

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For unipolar hydrodynamic model of semiconductor device represented by Euler-Poisson equations, when the doping profile is supersonic, the existence of steady transonic shock solutions and C^{∞}-smooth steady transonic solutions for Euler-Poisson Equations were established in [Lei-Mei-Zhang-Zhang 2018 SIMA] and [Wei-Mei-Zhang-Zhang 2022 SIMA], respectively. In this talk, we further study the nonlinear structural stability and the linear dynamic instability of these steady transonic solutions. When the C^1-smooth transonic steady-states pass through the sonic line, they produce singularities for the system, and cause some essential difficulty in the proof of structural stability. For any relaxation time: $0 < \tau \leq +\infty$, by means of elaborate singularity analysis, we first investigate the structural stability of the C^1-smooth transonic steady-states, once the perturbations of the initial data and the doping profiles are small enough. Moreover, when the relaxation time is large enough $\tau \gg 1$, under the condition that the electric field is positive at the shock location, we prove that the transonic shock steady-states are structurally stable with respect to small perturbations of the supersonic doping profile. Furthermore, we show the linearly dynamic instability for these transonic shock steady-states provided that the electric field is suitable negative. The proofs for the structural stability results are based on singularity analysis, a monotonicity argument on the shock position and the downstream density, and the stability analysis of supersonic and subsonic solutions. The linear dynamic instability of the steady transonic shock for Euler-Poisson equations can be transformed to the ill-posedness of a free boundary problem for the Klein-Gordon equation. By using a nontrivial transformation and the shooting method, we prove that the linearized problem has a transonic shock solution with exponential growths. These results enrich and develop the existing studies.