PIMS-AMI Workshop on Applied Harmonic Analysis and Statistical Learning

August 2-3, 2018

University of Alberta, Canada

Conference Venue

Venue: CAB 373, North Campus, University of Alberta

Organizers

Bin Han, University of Alberta, bhan@ualberta.ca

Rong-Qing Jia, University of Alberta, rjia@ualberta.ca

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Conference Schedule

August 2, Thursday

9:30 - 12:00 Chair: Rong-Qing Jia

9:30 – 10:20 **Yuesheng Xu**,
Sparse Machine Learning

10:20 – 10:40 **Tea Break**,

10:40 – 11:30 **Ding-Xuan Zhou**,
Mathematical Theory of Deep Convolutional Neural Networks

11:30 – 12:00 **Chenzhe Diao**,
Quasi-tight Framelets and Generalized Matrix Spectral Factorization of Laurent Polynomials

14:30 – 17:00 Chair: Bin Han

14:30 – 15:20 **Xiaosheng Zhuang**,
Affine Shear Tight Frames with 2-Layer Structure and Their Applications in Image/Video Processing

15:20 – 15:40 **Tea Break**,

15:40 – 16:30 **Yi Shen**,
A Novel Methodology for Removal of Periodic Noise Using Analysis Approach

16:30 – 17:00 **Michelle Michelle**,
On the Construction of Biorthogonal Wavelets on $[0, \infty)$

August 3, Friday

9:30 - 12:00 Chair: Ding-Xuan Zhou

9:30 – 10:20 **Rong-Qing Jia**,
Applications of Wavelet Analysis to Partial Differential Equations

10:20 – 10:40 **Tea Break**,

10:40 – 11:30 **Xikui Wang**,
Exploration versus Exploitation in Statistical Learning

11:30 – 12:00 **Ran Lu**,
The Oblique Extension Principle and Quasi-tight Framelets

14:30 – 17:00 Chair: Yuesheng Xu

14:30 – 15:20 **Jianbing Yang**,
Wavelet Frame Based Scattered Data Reconstruction

15:20 – 15:40 **Tea Break**,

15:40 – 16:30 **Bin Han**,
Tensor Product Complex Tight Framelets and Some Related Problems

16:30 – 17:00 **Bernal Manzanilla**,
Monogenic Signal and its Use for Patterns Detection

Quasi-tight Framelets and Generalized Matrix Spectral Factorization of Laurent Polynomials

Chenzhe Diao

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As a generalization of orthonormal wavelets in $L_2(\mathbb{R})$, tight framelets are of importance in wavelet analysis and applied sciences due to their many desirable properties in applications such as image processing and numerical algorithms. Tight framelets are often derived from particular refinable functions satisfying certain stringent conditions. Consequently, a large family of refinable functions cannot be used to construct tight framelets. This motivates us to introduce the notion of a quasi-tight framelet, which is a dual framelet but behaves almost like a tight framelet. It turns out that the study of quasi-tight framelets is intrinsically linked to the problem of the generalized matrix spectral factorization for matrices of Laurent polynomials. In this talk, we provide a systematic investigation on the generalized matrix spectral factorization problem and compactly supported quasi-tight framelets. As an application of our results on generalized matrix spectral factorization for matrices of Laurent polynomials, we show that from any arbitrary compactly supported refinable function in $L_2(\mathbb{R})$, we can always construct a compactly supported quasi-tight framelet having the minimum number of generators and the highest possible order of vanishing moments. Moreover, in multidimensional case, given an arbitrary dilation matrix M and a compactly supported M-refinable function $\phi \in L_2(\mathbb{R}^d)$, we can always derive a compactly supported quasi-tight M-framelet in $L_2(\mathbb{R}^d)$ with directionality or the highest possible order of vanishing moments.

Tensor Product Complex Tight Framelets and Some Related Problems

Bin Han

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This talk is based on several old papers on directional complex tight framelets. First we shall present some review on tight wavelet framelets and directional tensor product complex tight framelets. Next we shall mention their applications to image processing. Then we shall discuss several variations of complex tight framelets and several currently unresolved problems related to complex tight framelets. The comparison between tensor product complex tight framelets and dual tree complex wavelet transform will also be discussed.

Applications of Wavelet Analysis to Partial Differential Equations

Rong-Qing Jia

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In this talk we demonstrate that wavelet analysis can be used for both theoretical study and numerical computation of partial differential equations. In particular, we investigate smooth solutions of elliptic equations with non-smooth coefficients and establish global regularity of solutions of such equations. This result was not attainable by using traditional methods.

The Oblique Extension Principle and Quasi-tight Framelets

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It's well known in wavelet theory that the oblique extension principle (OEP) enables increasing the vanishing moments of high-pass filters derived from the given low-pass filters. However constructing multivariate tight framelets with high vanishing moments is known to be a challenging problem as it's related to the decomposition of Laurent polynomials into sum of squares. In this talk, we will introduce the concept of an OEP based quasi-tight framelet, which is similar but slightly weaker than a tight framelet. We will provide a method of constructing quasi-tight framelet filter banks with desired vanishing moments based on the OEP. Also we will further analyze the features on the derived high-pass filters, and make connections with the features on the low-pass filters.

Monogenic Signal and its Use for Patterns Detection

Bernal Manzanilla

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As technology such as machine learning progresses, pattern identification becomes critical. In the case of images, boundary detection and shape description is fundamental for object identification. Roughly speaking, the shape is extracted from changes in either pixel colors or reflection strength in the case of seismic images. The work carried out by applied mathematicians, physicists and engineers in the last couple of decades has explored ways to robustly extract the boundaries that define such changes in amplitude, or contrast, that allow (like in the case of the human eye to distinguish objects). In this talk, I will succinctly describe the use of monogenic signals as discrete functions that have analytic properties and its use to define geometric features and changes in images such as texture orientation and corners and edges, all of them, constituting a foundation for the task of identifying objects and patterns.

On the Construction of Biorthogonal Wavelets on $[0, \infty)$

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Many problems in sciences and engineering are defined on a bounded domain or even more simply a bounded interval. Even though wavelets have enjoyed many remarkable successes in providing numerous insights to such problems, current existing constructions of wavelets on $[0, \infty)$ and/or a bounded interval suffer from some serious shortcomings; e.g., reduced approximation order and vanishing moments nearby the boundary. Needless to say, these shortcomings bring forth negative impacts in applications of interest. In the context of numerical differential equations, for instance, the numerical solution may experience a reduced convergence rate to the true solution in the neighborhood of the boundary. In this talk, we propose a new simple method to construct biorthogonal wavelets on $[0, \infty)$, and showcase its ability in achieving the maximum polynomial reproduction ability and vanishing moments nearby the boundary. Several examples of this construction will be presented.

A Novel Methodology for Removal of Periodic Noise Using Analysis Approach

Yi Shen

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The theory of compressed sensing shows that it is highly possible to recover a sparse signal from few measurements. Due to its wide applications, compressed sensing has drawn attention of many researchers from the fields of signal and image processing, applied mathematics, and statistics. In this talk we are interested in signals which are sparse under redundant tight frames. Some sufficient conditions are provided to guarantee the stable recovery via solving analysis based approaches. Then the analysis approach is applied for digital removal of periodic noise.

Sparse Machine Learning

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We shall present recent mathematical development of sparse machine learning methods. In particular, we shall discuss recent results on learning in reproducing kernel Banach spaces.

Exploration versus Exploitation in Statistical Learning

Xikui Wang

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The bandit process is a statistical model of sequential learning and decision making. Sequential observations are made from several statistics populations, some or all of which have unknown statistical distributions. Each observation has two consequences. On one hand, it provides information about the distribution of the selected population. This is exploration and benefits later decisions because it helps reduce uncertainty when the population distribution is unknown. On the other hand, each observation defines a reward or cost. This is exploitation. The myopic strategy focuses on maximizing the expected value of the observation and ignores future observations. In reality, there is a tradeoff between exploration and exploitation in virtually every statistical learning problem. We discuss the models, methods and applications of bandit processes.

Wavelet Frame Based Scattered Data Reconstruction

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In real world applications many signals contain singularities, like edges in images. Recent wavelet frame based approaches were successfully applied to reconstruct scattered data from such functions while preserving these features. In this talk we present an approach which determines the approximant from shift invariant subspaces by minimizing an L_1-regularized least squares problem which makes additional use of the wavelet frame transform in order to preserve sharp edges. We give a detailed analysis of this approach, i.e., how the approximation error behaves dependent on data density and noise level. Moreover, we present some numerical examples, for instance how to apply this approach to handle coarse-grained models in molecular dynamics. This is a joint work with Prof. Zuowei Shen and Dominik Stahl at Department of Mathematics, National University of Singapore; and with Prof. Lanyuan Lu, Guanhua Zhu and Dudu Tong at School of Biological Sciences, Nanyang Technological University.

Mathematical Theory of Deep Convolutional Neural Networks

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Deep learning has been widely applied and brought breakthroughs in speech recognition, computer vision, and many other practical domains. But there lacks a mathematical foundation for deep learning, though the involved deep neural network architectures and computational issues have been studied in machine learning. In particular, there is no theoretical understanding for the approximation or generalization ability of deep learning methods such as deep convolutional neural networks. This talk describes a mathematical theory of deep convolutional neural networks (CNNs). In particular, we discuss the universality of a deep CNN, meaning that it can be used to approximate any continuous function to an arbitrary accuracy when the depth of the neural network is large enough. Our quantitative estimate, given tightly in terms of the number of free parameters to be computed, verifies the efficiency of deep CNNs in dealing with large dimensional data. Some related distributed learning algorithms will also be discussed.

Affine Shear Tight Frames with 2-Layer Structure and Their Applications in Image/Video Processing

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In this talk, we present the characterizations, construction, and applications of affine shear tight frames with 2-layer structure. First, we introduce affine shear systems with 2-layer structure that have generators splitting the frequency region at each scale into inner and outer layers. Second, we provide the characterizations of affine shear systems with 2-layer structure to be affine shear tight frames with 2-layer structure. Finally, we show that digital affine shear transforms with 2-layer structure can be implemented with low redundancy rate and with near-linear computational complexity. Numerical experiments are conducted to demonstrate the advantages of our transforms in image/video processing such as denoising and inpainting.