Math 421 Q1 Winter 2017 Homework 8 Solutions

Due Mar. 30, 12pm.

Total 20 points

QUESTION 1. (5 PTS) *Prove that in any simple graph, there are two vertices having the same degrees.*

QUESTION 2. (5 PTS) *Let $G = (V, E)$ with $V = 10$ and $E = 26$. Prove that there is a vertex in G whose degree is no less than 6.*

QUESTION 3. (5 PTS) *A sequence $(d_1, ..., d_n)$ is called "graphic" if and only if it is the degree sequence of some simple graph. Prove that the sequence $(6, 6, 5, 4, 2, 2, 1)$ is not graphic.*

QUESTION 4. (5 PTS) *Prove that in any group of 10 people there are either four mutual friends or three mutual strangers (non-friends, see last example of Mar. 22 lecture notes if you don't understand the problem).*