

## MATH 421 Q1 WINTER 2017 HOMEWORK 5

Due Mar. 2, 12pm.

Total 20 points

QUESTION 1. (5 PTS) Let  $a_n$  be the number of  $n$ -digit numbers formed by 1,3,5,7,9 with 3 and 7 appearing an even number of times. Find a formula for  $a_n$  using exponential generating functions.

QUESTION 2. (5 PTS) Use exponential generating function to find the number of ways color 6 pillars in a line with 4 colors ( $R, G, B, Y$ ) such that the number of pillars colored  $R$  is odd and the number of pillars colored  $G$  is even. **Give your answer in numerical value.**

QUESTION 3. (5 PTS) Let  $a_n$  be defined through  $a_{n+2} = 2a_{n+1} - a_n$  for all  $n \geq 0$  and  $a_0 = 0, a_1 = 1$ . Use generating functions to derive the formula for  $a_n$ .

QUESTION 4. (5 PTS) Let  $a_n$  satisfy  $a_{n+1} = 2a_n + n$  for all  $n \geq 0$  and  $a_0 = 1$ . Use generating functions to find the general formula for  $a_n$ .