Homework 9: Gauss-Bonnet

(Total 20 pts; Due Dec. 2 12pm)

Question 1. (5 pts) *Let S be a regular, orientable, compact surface with positive Gaussian curvature: $K > K_{\min} > 0$. Prove that the surface area of S is less than $4\pi / K_{\min}$.*

Question 2. (5 pts) *Let S be a compact oriented surface that can be smoothly deformed into a sphere. Let γ be a simple closed geodesic separating S into two regions A, B. Let $\mathcal{G}$: $S \mapsto \mathbb{S}^2$ be the Gauss map. Prove that $\mathcal{G}(A)$ and $\mathcal{G}(B)$ have the same area.*

Question 3. (5 pts) *Let S be a developable surface. Let γ be a curve on S. Let $\tilde{\gamma}$ be the "flattened" curve corresponding to γ on the plane that is the "flattened" S. Prove or disprove: The geodesic curvature of γ and the signed curvature of $\tilde{\gamma}$ are the same at corresponding points.*

Question 4. (5 pts) *Let $f: S_1 \mapsto S_2$ be a local isometry. Let a curve $\gamma_1 \subset S_1$ and $\gamma_2 := f(\gamma_1)$. Let $w_1(s)$ be a tangent vector field that is parallel along γ_1. For every $p \in \gamma_1$, Let $w_2(f(p)) := (Df)(p)(w_1(p))$. Then $w_2(s)$ is a tangent vector field along γ_2. Prove or disprove: w_2 is parallel along γ_2.*

The following are more abstract or technical questions. They carry bonus points.

There is no bonus problem for this homework.