

HOMWORK 7: PARALLEL TRANSPORT AND GEODESICS

(Total 20 pts; Due Nov. 7 12pm)

QUESTION 1. (5 PTS) *Let γ be a curve on S . Let w be a tangent vector field parallel along γ . Find all $\lambda: \gamma \mapsto \mathbb{R}$ such that λw is still parallel along γ .*

QUESTION 2. (5 PTS) *Let γ be a curve on S . Let w, \tilde{w} be unit vector fields along γ . Further assume that at every $p \in \gamma$, there holds the angle between w, \tilde{w} , $\angle(w, \tilde{w}) = \theta_0$, a constant. Prove or disprove: w is parallel along γ if and only if \tilde{w} is parallel along γ .*

QUESTION 3. (10 PTS) *Let S be a surface parametrized by $\sigma(u, v) = (u, v, uv)$.*

- a) (7 PTS) *Calculate the Christoffel symbols $\Gamma_{11}^1, \dots, \Gamma_{22}^2$.*
- b) (2 PTS) *Write down the geodesic equations for this surface.*
- c) (1 PT) *Prove that $u = \text{constant}$ and $v = \text{constant}$ are geodesics.*

The following are more abstract or technical questions. They carry bonus points.

There is no bonus question for this homework.