Homework 4: The First Fundamental Form

(Total 20 pts + bonus 5 pts; Due Oct. 14 12pm)

Question 1. (5 pts) *Calculate the first fundamental form of the surface*

$$\sigma(u,v) = (3\sin u\cos v, 2\sin u\sin v, \cos u). \tag{1}$$

Question 2. (5 pts) *Consider the surface patch* $\sigma(u,v) = (u\cos v, u\sin v, \ln(\cos v) + u)$. *Let* $u_1 < u_2$ *be arbitrary. Show that the arc length of the curve* $\sigma(t,v)$ *between* $t = u_1$ *and* $t = u_2$ *is independent of* v.

Question 3. (10 pts) *Let the first fundamental form for a surface patch be* $\mathrm{d}u^2 + (1 + u^2)\,\mathrm{d}v^2$.

a) (8 pts) *Calculate the lengths of the three sides and the three angles of the curvilinear triangle bounded by images of* $u = \frac{v^2}{2}$, $u = -\frac{v^2}{2}$, $v = 1$.

b) (2 pts) *Prove that the area of the curvilinear triangle is greater than* $1/3$.

The following are more abstract or technical questions. They carry bonus points.

Question 4. (**Bonus, 5 pts**) *Consider the surface of revolution* $\sigma(u,v) = (f(u)\cos v, f(u)\sin v, u)$ *where* $f(u) > 0$ *and* $v \in [0, 2\pi]$.

a) (2 pts) *Prove that it can always be parametrized so that the first fundamental form becomes* $\mathbb{E}(v)\,\mathrm{d}u^2 + \mathrm{d}v^2$.

b) (2 pts) *Find a conformal mapping between* $\sigma(u,v)$ *and the plane.*

c) (1 pt) *For what* f *is such a surface developable? Justify your claim.*