Math 334 Fall 2011: Summary of Quiz 4

Sep. 30, 2011

Solution and Grading Scheme.

- *Problem*: Solve
$$3\,y''+5\,y'+2\,y=0, \qquad y(0)=0,\ \ y'(0)=1. \tag{1}$$
- *Solution*: This is intial value problem, the universal procedure is to first find the general solution, then use the initial conditions to fix the constants.
    - General solution.
      First write down the characteristic equation:
      $$3\,r^2+5\,r+2=0 \Longrightarrow r_{1,2}=\frac{-5\pm\sqrt{5^2-4\cdot 3\cdot 2}}{6}=\frac{-5\pm 1}{6}=-\frac{2}{3},-1. \tag{2}$$
      So the general solution is
      $$y=C_1\,e^{-\frac{2}{3}t}+C_2\,e^{-t}. \tag{3}$$
    - Use the initial conditions.
      Preparation:
      $$y'=-\frac{2}{3}\,C_1\,e^{-\frac{2}{3}t}-C_2\,e^{-t}. \tag{4}$$
      Now
      $$y(0)=0 \Longrightarrow C_1+C_2=0; \tag{5}$$
      $$y'(0)=1 \Longrightarrow -\frac{2}{3}\,C_1-C_2=1. \tag{6}$$
      It is easy to see that $C_1=3, C_2=-3$.
    - Final answer:
      $$y=3\,e^{-\frac{2}{3}t}-3\,e^{-t}. \tag{7}$$
- *Grading Scheme*:
    - Know the overall procedure: General solution -> Use initial conditions. 2 pts;
    - Finding the general solution: 2 pt.
        - 1 pt for correct characteristic equation;
        - 1 pt for correct general solution.
    - Final answer: 1 pt.

Statistics.

5	4	3	2	1	0	Total
10	2	2	1	0	0	15

Table 1. Grade distribution

Popular Mistakes.

- Wrong characteristic equation.
  $y'=r\,e^{rt}$, $y''=e^{rt}+r^2\,e^{rt}$. The differentiation is with respect to t, not r. So r is just a "constant here". So $(r\,e^{rt})'=r\,(e^{rt})'=r^2\,e^{rt}$.
- Not careful enough(?):
$$\frac{-5\pm\sqrt{1}}{6}=-5/6+1 \text{ or } -5/6-1. \tag{8}$$
$$(3\,r+2)\,(r+1)=0, \qquad r=-1, -3/2. \tag{9}$$

- Wrong factorization
$$[3\,r^2+5\,r+2=](3\,r+1)\,(r+2). \tag{10}$$
- Forget to replace t by $t_0=0$:
$$\text{Then } C_1=3\ldots(1) \text{ is } 3\,e^{-\frac{2}{3}t}+C_2\,e^{-t}=0. \tag{11}$$

Some Suggestions.