Math 334 Fall 2011: Summary of Quiz 2

September 16, 2011

Solution and Grading Scheme.

- *Problem*: Solve
$$x\,(x^2+y^2)\,\mathrm{d}x + y\,(x^2+y^2)\,\mathrm{d}y = 0 \tag{1}$$
- *Solution*: We have
$$M(x,y) = x\,(x^2+y^2), \qquad N(x,y) = y\,(x^2+y^2). \tag{2}$$
Check
$$\frac{\partial M}{\partial y} = 2\,x\,y, \qquad \frac{\partial N}{\partial x} = 2\,x\,y. \tag{3}$$
So the equation is exact.
Compare
$$\int x\,(x^2+y^2)\,\mathrm{d}x \text{ and } \int y\,(x^2+y^2)\,\mathrm{d}y \tag{4}$$
we see that they are of exactly the same difficulty.
Write
$$u(x,y) = \int x\,(x^2+y^2)\,\mathrm{d}x + g(y) = \frac{1}{4}\,x^4 + \frac{1}{2}\,x^2\,y^2 + g(y). \tag{5}$$
Compute
$$\frac{\partial u}{\partial y} = 2\,x^2\,y + g'(y) \tag{6}$$
and compare with
$$N(x,y) = y\,(x^2+y^2) \tag{7}$$
we see that $g'(y) = y^3$ which gives $g(y) = y^4/4$.
So finally the general solution is given by
$$\frac{x^4}{4} + \frac{x^2\,y^2}{2} + \frac{y^4}{4} = C \tag{8}$$
or equivalently
$$(x^2+y^2)^2 = C \tag{9}$$
or even simpler
$$x^2+y^2 = C. \tag{10}$$
- *Grading Scheme*:
  - Know how to solve: 2 pts;
  - Correct integration: 1 pt.
  - Correct $g(y)$ (or $g(x)$): 1 pt;
  - Correct final answer: 1 pt.

Statistics.

5	4	3	2	1	0	Total
10	2	3	0	3	1	19

Table 1. Grade distribution

Popular Mistakes.

- Forgot to write down solution after getting $u(x,y)$.
- $x^2\,y + g'(y) = y\,(x^2+y^2) \Longrightarrow g'(y) = y^2$.
- $y\,(x^2+y^2)\,\mathrm{d}y = -x\,(x^2+y^2)\,\mathrm{d}x \Longrightarrow \int\, y\,(x^2+y^2)\,\mathrm{d}y = -\int\, x\,(x^2+y^2)\,\mathrm{d}x$. Such operation is only correct when doing a separable equation.

Some Suggestions.

- Some of you didn't check whether the equation is "exact" and just start solving it. When the equation is not exact, time can be wasted by doing this.