

MATH 334 FALL 2011 HOMEWORK 9

BASIC

Problem 1. Use definition to compute $\mathcal{L}\{\sin bt\}$.

Problem 2. Compute

$$\mathcal{L}\{e^{-t}t \sin 2t\}(s). \quad (1)$$

Problem 3. Compute the following inverse transforms.

a) $\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+2s+10}\right\}$.

b) $\mathcal{L}^{-1}\left\{\frac{3}{(2s+5)^3}\right\}$.

c) $\mathcal{L}^{-1}\left\{\frac{s-1}{2s^2+s+6}\right\}$.

Problem 4. Use Laplace transform to solve the following problem:

a) $y'' + 3y' + 2y = 0; \quad y(0) = 1, y'(0) = 0;$

b) $y^{(4)} - 4y''' + 6y'' - 4y' + y = 0; \quad y(0) = 0, y'(0) = 1, y''(0) = 0, y'''(0) = 1$

c) $y'' - 2y' + 2y = \cos t; \quad y(0) = 1, y'(0) = 0.$

INTERMEDIATE

ADVANCED

Problem 5. Compute

$$\mathcal{L}^{-1}\left\{\ln\left(\frac{s+2}{s-5}\right)\right\}. \quad (2)$$

CHALLENGE

Problem 6. Solve

$$y'' + 3ty' - 6y = 1, \quad y(0) = 0, \quad y'(0) = 0. \quad (3)$$

Problem 7. Solve

$$x' = 3x - 2y, \quad x(0) = 1; \quad (4)$$

$$y' = 3y - 2x; \quad y(0) = 1. \quad (5)$$

Answers:

- Problem 2: $\frac{4(s+1)}{[(s+1)^2+4]^2}$.
- Problem 3:
 - a) $e^{-t} \cos 3t$.
 - b) $\frac{3}{16} e^{-\frac{5}{2}t} t^2$.
 - c) $\frac{1}{2} e^{-\frac{1}{4}t} \cos\left(\frac{\sqrt{47}}{4}t\right) - \frac{5}{2\sqrt{47}} e^{-\frac{1}{4}t} \sin\left(\frac{\sqrt{47}}{4}t\right)$.
- Problem 4:
 - a) $2e^{-t} - e^{-2t}$.
 - b) $te^t - t^2e^t + \frac{2}{3}t^3e^t$.
 - c) $\frac{1}{5}\cos t - \frac{2}{5}\sin t + \frac{4}{5}e^t\cos t - \frac{2}{5}e^t\sin t$.
- Problem 5: $(e^{5t} - e^{-2t})/t$.
- Problem 6: $\frac{1}{2}t^2$.
- Problem 7: $x = y = e^t$.