Math 334 A1 Homework 3 (Due Nov. 5 5pm)

- No "Advanced" or "Challenge" problems will appear in homeworks.

Basic Problems

Problem 1. (4.1 11) Verify that the given functions are solutions of the differential equation, and determine their Wronskian.

$$y''' + y' = 0; \qquad 1,\ \cos t,\ \sin t. \tag{1}$$

Problem 2. (4.2 1) Express $1 + i$ in the form $R(\cos\theta + i \sin\theta) = R\, e^{i\theta}$.

Problem 3. (4.2 9) Find all four roots of $1^{1/4}$.

Problem 4. (5.1 7) Determine the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^2 (x+2)^n}{3^n}. \tag{2}$$

Problem 5. (5.1 13) Determine the Taylor series about x_0 for the given function:

$$y(x) = \ln x, \qquad x_0 = 1. \tag{3}$$

Problem 6. (5.1 21) Rewrite the given expression as a sum whose generic term involves x^n:

$$\sum_{n=2}^{\infty} n(n-1)\, a_n\, x^{n-2}. \tag{4}$$

Problem 7. (5.2 3) Consider

$$y'' - x\, y' - y = 0, \qquad x_0 = 1, \tag{5}$$

a) Find the first four terms in each of two solutions y_1 and y_2 (unless the series terminates sooner).

b) By evaluating the Wronskian $W(y_1, y_2)(x_0)$, show that y_1 and y_2 form a fundamental set of solutions (that is y_1, y_2 are linearly independent.)

Problem 8. (5.2 15) Find the first five nonzero terms in the solution of the problem

$$y'' - x\, y' - y = 0, \qquad y(0) = 2, \quad y'(0) = 1. \tag{6}$$

Problem 9. (5.3 7) Determine a lower bound for the radius of convergence of series solutions about each given point x_0 for the differential equation

$$\left(1 + x^3\right) y'' + 4\, x\, y' + 4\, y = 0; \qquad x_0 = 0; \quad x_0 = 2. \tag{7}$$

Problem 10. (5.3 12) Find the first four nonzero terms in each of two power series solutions about the origin for

$$e^x\, y'' + x\, y = 0 \tag{8}$$

Determine the lower bound of radius of convergence.

Problem 11. (5.4 1) Find the general solution

$$x^2\, y'' + 4\, x\, y' + 2\, y = 0. \tag{9}$$

Problem 12. (5.4 19) Find all singular points of

$$x^2 (1 - x)\, y'' + (x - 2)\, y' - 3\, x\, y = 0, \tag{10}$$

and determine whether each one is regular or irregular.

Intermediate Problems

Problem 13. (4.1 8) Determine whether the given set of functions is linearly dependent or linearly independent. If they are linearly dependent, find a linear relation among them.

$$f_1(t) = 2t - 3, \qquad f_2(t) = 2t^2 + 1, \qquad f_3(t) = 3t^2 + t. \tag{11}$$

(Note: As f_1, f_2, f_3 are not solutions to some 3rd order equation, Wronskian $\neq 0$ implies linear independence, but Wronskian $= 0$ does not imply linear dependence. Finding a "linear relation" means finding constants C_1, C_2, C_3 such that

$$C_1\, f_1 + C_2\, f_2 + C_3\, f_3 = 0. \tag{12}$$

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Problem 14. (4.2 11) Find the general solution of

$$y''' - y'' - y' + y = 0. \tag{13}$$

Problem 15. (4.2 16) Find the general solution of

$$y^{(4)} - 5\, y'' + 4\, y = 0. \tag{14}$$