Math 334 A1 Homework 2 (Due Oct. 8 5pm)

- No "Advanced" or "Challenge" problems will appear in homeworks.

Basic Problems

Problem 1. (1.1 1) Consider

$$y' = 3 - 2\,y. \tag{1}$$

Determine the behavior of y as $t \longrightarrow \infty$ by drawing the direction field and analyze it.

Problem 2. (1.3 27) Let

$$u_1(x, t) = \sin \lambda\, x \sin \lambda\, a\, t, \qquad u_2(x, t) = \sin(x - a\, t) \tag{2}$$

where λ is a real constant. Verify that u_1, u_2 are solutions of

$$a^2\, u_{xx} = u_{tt}. \tag{3}$$

Problem 3. (Ch.2 1) Solve

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^3 - 2\,y}{x}. \tag{4}$$

Problem 4. (Ch.2 3) Solve

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\,x + y}{3 + 3\,y^2 - x}, \qquad y(0) = 0. \tag{5}$$

Problem 5. (Ch.2 22) Solve

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^2 - 1}{y^2 + 1}, \qquad y(-1) = 1. \tag{6}$$

Problem 6. (3.1 1) Find the general solution:

$$y'' + 2\,y' - 3\,y = 0. \tag{7}$$

Problem 7. (3.1 9) Solve

$$y'' + y' - 2\,y = 0, \qquad y(0) = 1, \quad y'(0) = 1. \tag{8}$$

Problem 8. (3.3 7) Find the general solution

$$y'' - 2\,y' + 2\,y = 0. \tag{9}$$

Problem 9. (3.4 11) Solve

$$9\,y'' - 12\,y' + 4\,y = 0, \qquad y(0) = 2, \quad y'(0) = -1. \tag{10}$$

Intermediate Problems

Problem 10. (2.2 31) Solve

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^2 + x\,y + y^2}{x^2}. \tag{11}$$

Problem 11. (2.4 28) Solve

$$t^2\,y' + 2\,t\,y - y^3 = 0. \tag{12}$$

Problem 12. (3.1 17) Find a differential equation whose general solution is $y = c_1\,e^{2t} + c_2\,e^{-3t}$.