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Mathematical and Statistical Sciences
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Math 300, Advanced Boundary Value Problems I
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Review-Midterm

Exercise 1:
Solve the initial value problem for $-\infty < x < \infty$, $t \geq 0$

$$\begin{aligned} \frac{\partial w(x,t)}{\partial t} + 5\frac{\partial w(x,t)}{\partial x} &= e^{3t} \\ w(x,0) &= e^{-x^2} \end{aligned}$$

Exercise 2:
Solve the wave equation for $-\infty < x < \infty$, $t \geq 0$:

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= 25\frac{\partial^2 u}{\partial x^2} \\ u(x,0) &= x^2 \\ \frac{\partial u(x,0)}{\partial t} &= 3. \end{aligned}$$

Exercise 3:
Given

$$f(x) = \begin{cases} -1 & , x < -1 \\ x & , -1 < x < 1 \\ 1 & , x \geq 1 \end{cases}$$

(i) Find the Fourier series of $f(x)$ on $[-2,2]$.
(ii) Find the Fourier-sine series of $f(x)$ on $[0,2]$.
(iii) Find the Fourier-cosine series of $f(x)$ on $[0,2]$.

Exercise 4:
Study the following heat equation

$$\begin{aligned} \frac{\partial u}{\partial t} &= 13\frac{\partial^2 u}{\partial x^2}, \qquad 0 \leq x \leq 2 \\ \frac{\partial u(0,t)}{\partial x} = 0 & \qquad \frac{\partial u(2,t)}{\partial x} = 0 \\ u(x,0) &= f(x), \end{aligned}$$

where $f(x)$ is as given in Exercise 3.

(a) Find the time dependent solution. Show all steps which are necessary. You might use the result from Exercise 3.
(b) Find the asymptotic solution as $t \to \infty$.