MATH 300 Fall 2004
Advanced Boundary Value Problems I
Assignment 4
Due: Friday November 5, 2004

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Question 1. [p 205, #2]

Solve the vibrating membrane problem given below:

$$\frac{\partial^2 u}{\partial t^2} = 100\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right), \quad 0 < r < 1, \quad t > 0$$

$$u(a,t) = 0, \qquad t > 0$$

$$u(r,0) = 1 - r^2, \qquad 0 < r < 1$$

$$\frac{\partial u}{\partial t}(r,0) = 1, \qquad 0 < r < 1.$$

Question 2. [p 206, #4]

Solve the vibrating membrane problem given below:

$$\frac{\partial^2 u}{\partial t^2} = \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right), \quad 0 < r < 1, \quad t > 0$$

$$u(a,t) = 0, \qquad t > 0$$

$$u(r,0) = 0, \qquad 0 < r < 1$$

$$\frac{\partial u}{\partial t}(r,0) = J_0(\alpha_3 r), \qquad 0 < r < 1.$$

Question 3. [p 331, #2]

If $f(x)$ is an even function and $g(x)$ is an odd function, show that the set of functions $\{f(x), g(x)\}$ is orthogonal with respect to the weight function

$$w(x) = 1$$

on any symmetric interval $[-a, a]$ containing 0.

Question 4. [p 332, #6]

Show that the set of functions $\left\{1,\, 1 - x,\, \frac{1}{2}(2 - 4x + x^2)\right\}$ is orthogonal with respect to the weight function

$$w(x) = e^{-x}$$

on the interval $[0, \infty)$. (These are examples of **Laguerre polynomials**.)

Question 5. [p 332, #8]

Show that the set of functions $\left\{\frac{1}{2}(2-4x+x^2),\ -12x+8x^3\right\}$ is orthogonal with respect to the weight function

$$w(x) = e^{-x}$$

on the interval $[0,\infty)$.

Question 6. [p 344, #6]

Given the boundary value problem

$$y'' + \left(\frac{1+\lambda x}{x}\right) y = 0$$
$$y(1) = 0$$
$$y(2) = 0,$$

on the interval $[1,2]$, put the equation in Sturm-Liouville form and decide whether the problem is regular or singular.

Question 7. [p 344, #8]

Given the boundary value problem

$$(1-x^2)y'' - 2xy' + (1+\lambda x)y = 0$$
$$y(-1) = 0$$
$$y(1) = 0,$$

on the interval $[-1,1]$, put the equation in Sturm-Liouville form and decide whether the problem is regular or singular.

Question 8. [p 344, #14]

Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem

$$y'' + \lambda y = 0$$
$$y(-\pi) = 0$$
$$y(\pi) = 0$$
$$y'(-\pi) = y'(\pi).$$

Question 9. [p 344, #16]

Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem

$$y'' + \lambda y = 0$$
$$y(0) + y'(0) = 0$$
$$y(2\pi) = 0.$$

Question 10. [p 344, #22]

Show that the boundary value problem

$$y'' - \lambda y = 0$$
$$y(0) + y'(0) = 0$$
$$y(1) + y'(1) = 0$$

has one positive eigenvalue. Does this contrdict Theorem 1?