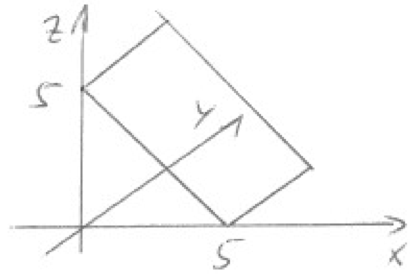
Assignment #4 Solutions

1. $z = 5-x$, $0 \le x \le 5$, $0 \le y \le 3$



$\iint_R (5-x)\,dA$ is the volume of a solid that lies below the plane $z = 5-x$ and above the rectangle $[0,5]\times[0,3]$.

Its volume is $\frac{1}{2}\cdot 5 \cdot 5 \cdot 3 = \frac{75}{2}$

Hence $\iint_R (5-x)\,dA = \frac{75}{2}$

2. $$\int_1^4 \int_0^2 (x+\sqrt{y})\,dx\,dy = \int_1^4 \left[\frac{x^2}{2} + \sqrt{y}\cdot x\right]_{x=0}^{x=2} dy$$

$$= \int_1^4 (2 + 2\sqrt{y})\,dy$$

$$= 2y + \frac{4}{3}y^{3/2}\Big|_1^4 = \frac{46}{3}$$

3. $$V = \int_{-1}^{1}\int_0^2 (4 + x^2 - y^2)\,dy\,dx$$

$$= \int_{-1}^{1} \left[4y + x^2 y - \frac{y^3}{3}\right]_{y=0}^{y=2} dx$$

$$= \int_{-1}^{1} \left(8 + 2x^2 - \frac{8}{3}\right) dx = \int_{-1}^{1} \left(\frac{16}{3} + 2x^2\right) dx$$

$$= \frac{16}{3}x + \frac{2}{3}x^3 \Big|_{-1}^{1} = 12$$

4. $A(R) = 4$

$$f_{ave} = \frac{1}{4} \int_0^4 \int_0^1 e^y \sqrt{x + e^y} \, dy \, dx$$

$$= \frac{1}{4} \int_0^4 \left[\frac{2}{3} (x + e^y)^{3/2} \right]_{y=0}^{y=1} dx$$

$$= \frac{1}{4} \int_0^4 \frac{2}{3} \left((x+e)^{3/2} - (x+1)^{3/2} \right) dx$$

$$= \frac{1}{6} \frac{2}{5} \left[(x+e)^{5/2} \Big|_0^4 - (x+1)^{5/2} \Big|_0^4 \right]$$

$$= \frac{1}{15} \left((4+e)^{5/2} - e^{5/2} - 5^{5/2} + 1 \right)$$

5. $$\int_0^1 \int_x^{2-x} (x^2 - y) \, dy \, dx = \int_0^1 \left[x^2 y - \frac{y^2}{2} \right]_{y=x}^{y=2-x} dx$$

$$= \int_0^1 x^2(2-x) - \frac{(2-x)^2}{2} - x^3 + \frac{x^2}{2} \, dx$$

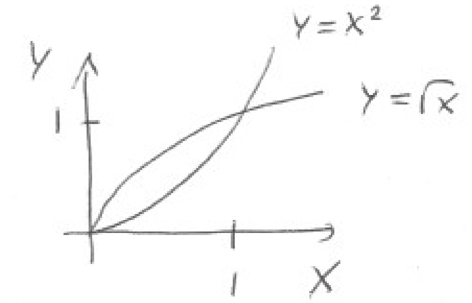
$$= \int_0^1 2x^2 - x^3 - 2 + 2x - \cancel{\frac{x^2}{2}} - x^3 + \cancel{\frac{x^2}{2}} \, dx$$

$$= \int_0^1 -2x^3 + 2x^2 + 2x - 2 \, dx$$

$$= -\frac{1}{2} x^4 + \frac{2}{3} x^3 + x^2 - 2x \Big|_0^1$$

$$= -\frac{5}{6}$$

6.



$D=\{(x,y)\mid 0\le x\le 1,\ x^2\le y\le\sqrt{x}\}$

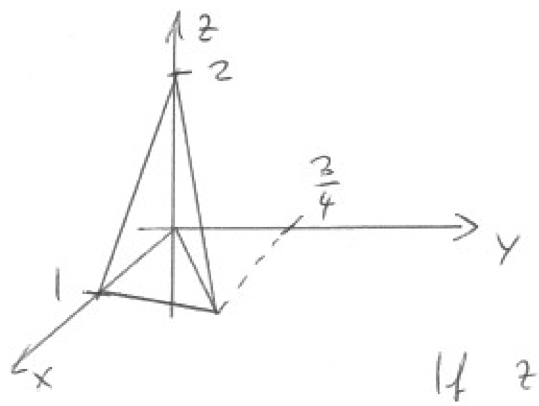
$$\int_0^1\int_{x^2}^{\sqrt{x}}(x+y)\,dy\,dx=\int_0^1\left[xy+\frac{y^2}{2}\right]_{y=x^2}^{y=\sqrt{x}}dx$$

$$=\int_0^1\left(\sqrt{x}^3+\frac{x}{2}-x^3-\frac{x^4}{2}\right)dx$$

$$=\frac{2}{5}x^{\frac{5}{2}}+\frac{x^2}{4}-\frac{x^4}{4}-\frac{x^5}{10}\Big|_0^1$$

$$=0.3$$

7.



$6x+2y+3z=6$:

$y=0,\ z=0\Rightarrow x=1$

$x=0,\ y=0\Rightarrow z=2$

$z=0,\ x=y\Rightarrow 8y=6$

$y=\frac{3}{4}$

If $z=0$ then $6x+2y=6$

or $x=1-\frac{1}{3}y$

$D=\{(x,y)\mid 0\le y\le\frac{3}{4}\mid y\le x\le 1-\frac{1}{3}y\}$

$6x+2y+3z=6\Rightarrow z=2-2x-\frac{2}{3}y$

$$Vol=\int_0^{3/4}\int_y^{1-\frac{1}{3}y}\left(2-2x-\frac{2}{3}y\right)dx\,dy$$

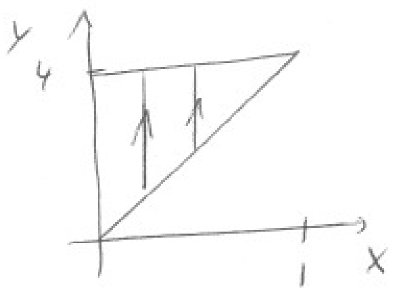
$$=\int_0^{3/4}\left[2x-x^2-\frac{2}{3}yx\right]_{x=y}^{x=1-\frac{1}{3}y}dy$$

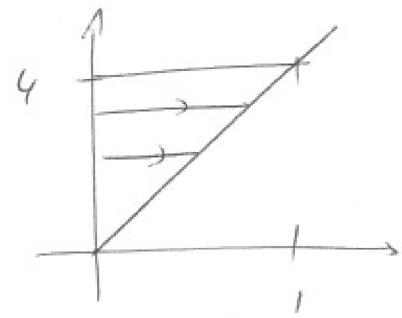
$$= \int_0^{3/4} \left[2 - \tfrac{2}{3}y - \left(1 - \tfrac{2}{3}y\right)^2 - \tfrac{2}{3}y\left(1 - \tfrac{1}{3}y\right) - 2y + y^2 + \tfrac{2}{3}y^2\right] dy$$

$$= \frac{1}{4}$$

8.

$$D = \{(x,y) \mid 0 \le x \le 1, \quad 4x \le y \le 4\}$$

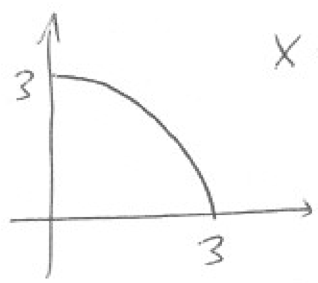




$$D = \{(x,y) \mid 0 \le y \le 4, \quad 0 \le x \le \tfrac{y}{4}\}$$

$$\int_0^1 \int_{4x}^4 f(x,y)\,dy\,dx = \int_0^4 \int_0^{y/4} f(x,y)\,dx\,dy$$

9.

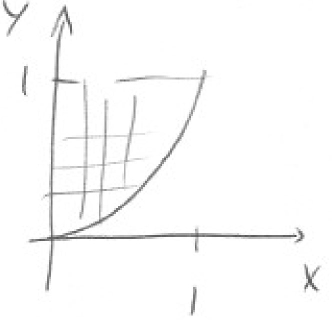


$$x = \sqrt{9-y^2} \quad \Rightarrow \quad x^2 = 9 - y^2$$
$$x = \sqrt{9-y^2}$$

$$D = \{(x,y) \mid 0 \le y \le 3, \quad 0 \le x \le \sqrt{9-y^2}\}$$
$$= \{(x,y) \mid 0 \le x \le 3, \quad 0 \le y \le \sqrt{9-x^2}\}$$

$$\int_0^3 \int_0^{\sqrt{9-y^2}} g(x,y)\,dx\,dy = \int_0^3 \int_0^{\sqrt{9-x^2}} g(x,y)\,dy\,dx$$

10.



$$D = \{(x,y) \mid 0 \le x \le 1,\ x^2 \le y \le 1\}$$
$$= \{(x,y) \mid 0 \le y \le 1,\ 0 \le x \le \sqrt{y}\}$$

$$\int_0^1 \int_{x^3}^1 x^3 \sin(y^3)\, dy\, dx = \int_0^1 \int_0^{\sqrt{y}} x^3 \sin(y^3)\, dx\, dy$$

$$= \int_0^1 \left[\frac{x^4}{4} \sin(y^3) \right]_{x=0}^{x=\sqrt{y}} dy$$

$$= \int_0^1 \frac{y^2}{4} \sin(y^3)\, dy$$

$$= -\frac{1}{4}\frac{1}{3}\left[\cos(y^3)\right]_0^1$$

$$= \frac{1}{12}\left(1 - \cos(1)\right)$$