Math 209

Assignment 8

Due: 12 Noon on Thursday, November 24, 2005.

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1. Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

   (a) $\int_C (y + e^{\sqrt{x}})dx + (2x + \cos y^2)dy$, C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

   (b) $\int_C \sin y \, dx + x \cos y \, dy$, C is the ellipse $x^2 + xy + y^2 = 1$.

2. If f is a harmonic function, that is $\nabla^2 f = 0$, show that the line integral $\int f_y dx - f_x dy$ is independent of path in any simple region D.

3. Find the area enclosed by the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$.

4. Let
$$I = \int_C \frac{ydx - xdy}{x^2 + y^2}$$
   where C is a circle oriented counterclockwise.

   (a) Show that $I = 0$ if C does not contain the origin.

   (b) What is I if C contain the origin?

5. Find the curl and the divergence of the vector field $\mathbf{F} = e^x \sin y \, \mathbf{i} + e^x \cos y \, \mathbf{j} + z \, \mathbf{k}$. Is $\mathbf{F}$ conservative?

6. Is there a vector field $\mathbf{G}$ on R^3 such that $\operatorname{curl} \mathbf{G} = xy^2 \, \mathbf{i} + yz^2 \, \mathbf{j} + zx^2 \, \mathbf{k}$? Explain.

7. Identify the surface with the given vector equation.

   (a) $\mathbf{r}(u, v) = u \cos v \, \mathbf{i} + u \sin v \, \mathbf{j} + u^2 \, \mathbf{k}$

   (b) $\mathbf{r}(x, \theta) = \langle x, x \cos\theta, x \sin\theta \rangle$

8. Find a parametric representation for the surface.

   (a) The part of an elliptic paraboloid $x + y^2 + 2z^2 = 4$ that lies in front of the plane $x = 0$

   (b) The part of a sphere $x^2 + y^2 + z^2 = 16$ that lies above the cone $z = \sqrt{x^2 + y^2}$

9. Find the area of the part of the surface $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

10. Find the area of the part of the surface $z = x^2 + 2y$ that lies above the triangle with vertices $(0, 0)$, $(1, 0)$, and $(1, 2)$.