Math 209

Assignment 5

Due: 12 Noon on Thursday, October 27, 2005.

1. Integrate $f(x,y) = \sin(\sqrt{x^2+y^2})$ over:
   (a) the closed unit disc;
   (b) the annular region $1 \leqslant x^2+y^2 \leqslant 4$.

2. Calculate the following integrals by changing to polar coordinates:

   (a) $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{x^2+y^2}\,dy\,dx$; (b) $\displaystyle\int_0^1 \int_{-\sqrt{x-x^2}}^{\sqrt{x-x^2}} \sqrt{x^2+y^2}\,dy\,dx$.

3. Find the area of the region inside the circle $r = 3\cos\theta$ and outside the cardioid $r = 1+\cos\theta$.

4. Find the volume of the solid bounded above by $z = 1-(x^2+y^2)$, bounded below by the xy–plane, and bounded on the sides by the cylinder $x^2+y^2-x = 0$.

5. Find the mass and centre of mass of the plate that occupies the given region Ω with the given density function λ.
   (a) $\Omega = \{(x,y) \in \mathbb{R}^2;\ 0 \leqslant x \leqslant a,\ 0 \leqslant y \leqslant \sqrt{a^2-x^2}\}$; $\lambda(x,y) = xy$.
   (b) Ω is the region inside the circle $r = 2\sin\theta$ and outside the circle $r = 1$; $\lambda(x,y) = y$.

6. Consider a square fan blade with sides of length 2 and the lower left corner placed at the origin. If the density of the blade is $\lambda(x,y) = 1 + x/10$, is it more difficult to rotate the blade about the x-axis or the y-axis?

7. Find the surface area of the surface $z = 1+3x+2y^2$ that lies above the triangle with vertices $(0,0)$, $(0,1)$ and $(2,1)$.

8. Find the surface area of the paraboloid $z = 4-x^2-y^2$ that lies above the xy-plane.

9. Find the surface area of the surface $z = \frac{2}{3}(x^{3/2}+y^{3/2})$ for $0 \leqslant x \leqslant 1$ and $0 \leqslant y \leqslant 1$.

10. Find the surface area of the sphere $x^2+y^2+z^2 = 4z$ that lies inside the paraboloid $z = x^2+y^2$.