Math 209

Assignment 2

Due: 12 Noon on Thursday, September 29, 2005.

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1. Let $R = \ln(u^2 + v^2 + w^2)$, $u = x + 2y$, $v = 2x - y$, and $w = 2xy$. Use the Chain Rule to find $\frac{\partial R}{\partial x}$ and $\frac{\partial R}{\partial y}$ when $x = y = 1$.

2. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $xyz = \sin(x + y + z)$.

3. Let f and g be two differentiable real valued functions. Show that any function of the form $z = f(x + at) + g(x - at)$ is a solution of the wave equation $\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$.

4. A function f is called **homogeneous of degree n** if it is satisfies the equation $f(tx, ty) = t^n f(x, y)$ for all t, where n is a positive integer. Show that if f is homogeneous of degree n, then
$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x, y)$$
[**Hint:** Use the Chain Rule to differentiate $f(tx, ty)$ with respect t.]

5. Find the directional derivative of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the point $(1, 2, -2)$ in the direction of vector $\mathbf{v} = \langle -6, 6, -3 \rangle$.

6. The temperature at a point (x, y, z) on the surface of a metal is $T(x, y, z) = 200e^{-x^2-3y^2-9z^2}$ where T is measured in degree Celsius and x, y, z in meters.
   (a) In which direction does the temperature increase fastest at the point $P(2, -1, 2)$?
   (b) What is the maximum rate of change at $P(2, -1, 2)$?

7. Find the points on the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ where the tangent plane is parallel to the plane $3x - 2y + 3z = 1$.

8. Find the local maximum and minimum values and saddle point(s) of the function
$$f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2.$$

9. Find the points on surface $x^2y^2z = 1$ that are closest to the origin.

10. Find the extreme values of $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ on the region
$$D = \{(x, y) |\ x^2 + y^2 \leq 16\}.$$