Math 209

Assignment 10

Not to be handed in.

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1. Evaluate $\displaystyle\iint_S xyz\,d\sigma$, where S is the portion of the surface $x^2+z^2=4$ in the first octant between the planes $y=0$ and $y=1$. $\boxed{\textit{Ans. } 2.}$

2. Show that the area cut from the surface $az=y^2-x^2$ by the cylinder $x^2+y^2=a^2$ is $(5\sqrt{5}-1)\pi a^2/6$.

3. A thin metal funnel has the shape of the part of the cone $z=\sqrt{x^2+y^2}$ between $z=1$ and $z=5$. Find the total mass of the funnel if its density (mass per unit area) is given by $\lambda(x,y,z)=x+z$. $\boxed{\textit{Ans. } \frac{248\sqrt{2}}{3}\pi.}$

4. Use the divergence theorem to find the total flux out of the given solid.

   (a) $\vec{v}(x,y,z)=(2xy+2z)\,\vec{i}+(y^2+1)\,\vec{j}-(x+y)\,\vec{k}$;

   where the solid occupies $0\leqslant x\leqslant 4,\quad 0\leqslant y\leqslant 4-x,\quad 0\leqslant z\leqslant 4-x-y.$ $\boxed{\textit{Ans. } \frac{2^7}{3}.}$

   (b) $\vec{v}(x,y,z)=2x\,\vec{i}+xy\,\vec{j}+xz\,\vec{k}$; where the solid occupies $x^2+y^2+z^2\leqslant 4$. $\boxed{\textit{Ans. } \frac{64}{3}\pi.}$

5. The sphere $x^2+y^2+z^2=a^2$ intersects the plane $x+2y+z=0$ in a curve C. Calculate $\displaystyle\oint_C \vec{v}\cdot d\vec{r}$, where $\vec{v}=2y\,\vec{i}-z\,\vec{j}+2x\,\vec{k}$ by using Stokes' theorem. $\boxed{\textit{Ans. } \pm\frac{5}{\sqrt{6}}\pi a^2.}$

6. The cylinder $x^2+y^2=b^2$ intersects the plane $y+z=a$ in a curve C. Calculate $\displaystyle\oint_C \vec{v}\cdot d\vec{r}$, where $\vec{v}=xy\,\vec{i}+yz\,\vec{j}+xz\,\vec{k}$, by using Stokes' theorem. $\boxed{\textit{Ans. } \pm\pi ab^2.}$

7. Evaluate $\displaystyle\iint_S \vec{F}\cdot\vec{n}\,d\sigma$, where $\vec{F}=\langle z^2-x,-xy,3z\rangle$ and S is the surface of the region bounded by $z=4-y^2$, $x=0$, $x=3$ and the xy-plane. $\boxed{\textit{Ans. } 16.}$