Math 209

Assignment 1

<u>Due</u>: 12 Noon on Thursday, September 22, 2005.

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1. Identify and sketch the level surface $f(x, y, z) = 1$ for the function $f(x, y, z) = z^2 - 36x^2 - 9y^2$.

2. Find the limit if it exists, or show that the limit does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x+y)^2}{x^2+y^2}$.

3. Find the limit if it exists, or show that the limit does not exist: $\displaystyle\lim_{(x,y)\to(1,-1)} \frac{x^2+y^2-2x-2y}{x^2+y^2-2x+2y+2}$.

4. Find the limit if it exists, or show that the limit does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{\sqrt{x^2+y^2}}$.

5. Find the limit if it exists, or show that the limit does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4+y^4}{(x^2+y^2)^{3/2}}$.

6. Find the limit if it exists, or show that the limit does not exist: $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^2+z^2}$.

   <u>Hint</u>. Consider using spherical coordinates: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$.

7. The partial differential equation
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$
   is called *Laplace's Equation*, named after the eminent French mathematician Pierre Simon de Laplace (1749 — 1827). Solutions of this equation are called *harmonic functions* and play a role in problems of heat conduction, fluid flow, and electric potential. Which of the following functions are solutions of Laplace's equation?

   (a) $u(x, y) = x^2 + y^2$;

   (b) $u(x, y) = \ln(x^2 + y^2)^{3/2}$;

   (c) $u(x, y) = \sin x \cosh y + \cos x \sinh y$.

8. Find an equation of the tangent plane to $z = e^x \ln y$ at $(3, 1, 0)$.

9. Find the differential of the following function: $w = \dfrac{x+y}{y+z}$.

10. Use differentials to estimate the amount of metal in a closed cylindrical can that is 10 centimeters high and 4 centimeters in diameter if the metal in the wall is 0.05 centimeters thick and the metal in the top and bottom is 0.1 centimeters thick.