7.7.9 Separation $u(r,\theta,t) = R(r)\,\Theta(\theta)\,T(t)$ leads to

θ: $\left.\begin{array}{l} \Theta'' = -\tau\,\Theta \\ \Theta(0)=0,\ \Theta(\pi)=0 \end{array}\right\} \Rightarrow \begin{array}{l} \tau_m = m^2 \qquad m=1,2,3,\ldots \\ \Theta_m(\theta) = \sin(m\theta) \end{array}$

r: $\left.\begin{array}{l} r(rR')' + (\lambda r^2 - m^2)R = 0 \\ R'(a)=0,\quad |R(0)|<\infty \end{array}\right\}$ Bessel equation of order m. No Y_m because of $|R(0)|<\infty$.

$$R_m(r) = J_m(\sqrt{\lambda}\,r) \qquad R_m'(a) = 0 = J_m'(\sqrt{\lambda}\,a)$$

Let w_{mn} denote the sequence of maxima and minima of $J_m(w)$ in increasing order. Then

$$\sqrt{\lambda_{mn}}\,a = w_{mn} \qquad \lambda_{mn} = \left(\frac{w_{mn}}{a}\right)^2.$$

t: ~~$T(t) = a_{mn}\cos(c\sqrt{\lambda_{mn}}\,t) + b_{mn}\sin(c\sqrt{\lambda_{mn}}\,t)$~~

Sorry, we have heat eq:

~~Superposition~~

~~$u(r,\theta,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\cos(c\sqrt{\lambda_{mn}}\,t) + b_{mn}\sin$~~

$$T(t) = a_{mn}\,e^{-k\lambda_{mn}t}$$

Superposition:

$$u(r,\theta,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}\,e^{-k\lambda_{mn}t}\,J_m(\sqrt{\lambda_{mn}}\,r)\sin(m\theta)$$