

(4.6) Strong solutions

Theorem 1 $n=2$: $u_0 \in V$, $f \in L^2_{loc}(0, \infty; H)$

then $\frac{du}{dt} + \nu Au + B(u, u) = f$ is a equality in $L^2(0, T; H)$

and $u \in L^\infty(0, T; V) \cap L^2(0, T; D(A))$ unique.

Moreover $u \in C^0([0, T]; V)$ and $u_0 \mapsto u(t)$ is continuous.

Proof: [R. p. 252 ff]

If f does not depend on time, then

Semidynamical system $(V, \{S_V(t)\}_{t \geq 0})$.

Theorem 2 $n=3$: Strong solutions are unique.