

(4.5) Unique weak solutions in 2-D

Theorem 1: $n=2$. The solution, u , of the Nav-Stokes eq

$$\frac{du}{dt} + \nu Au + B(u, u) = f$$

satisfies $u \in C^0([0, T]; H)$

and $u_0 \mapsto u(t)$ is continuous. In particular:

The solution is unique.

Proof: In class presentation #5.

Consequence: Lipschitz property

$$\|u(t) - v(t)\|_{L^2} \leq L(T) \|u_0 - v_0\|_{L^2} \quad 0 \leq t \leq T.$$

Semidynamical system $(H, \{S_H(t)\}_{t \geq 0})$

$S_H(t)u_0 = u(t)$, $S_H(t)$ is a C^0 -semigroup.