~~$\lambda_{1/2} = \pm i\beta$ $\beta < 1$ stable "spiral"~~

~~**Note.** $|\lambda_i| =$ Lyapunov multipliers.~~

(1.3) Connection of discrete and continuous

a) Poincaré map of periodic orbits $\longrightarrow$ later

b) Time-one map.

linear ODE $\dot{x} = Ax$, $A \in \mathbb{R}^{n \times n}$

general solution to initial condition $x(0) = x_0$:

$x(t) = e^{tA} x_0$, flow $T(t) = e^{tA}$.

We observe this flow in stroboscopic light, eg each second

$$x_1 = e^{A} x_0, \quad x_2 = e^{A} x_1, \quad \ldots \quad \underbrace{x_{n+1} = e^{A} x_n}_{\text{this defines}}$$

a discrete dynamical system $x_{n+1} = e^{A} x_n$

Stability of the trivial equilibrium $\bar{x} = 0$:

Theorem 2:

(a) μ eigenvalue of A $\Rightarrow$ $\lambda = e^{\mu}$ eigenvalue of e^A.

(b) $\mathrm{Re}\,\mu < 0$ $\Leftrightarrow$ $|\lambda| < 1$.

Proof: Homework!

Rule: "Lyapunov number $= e^{\text{Lyapunov exponent}}$"

(1.4) Abel's formula, the Wronskian

Linear ODE in $\mathbb{R}^n$, nonautonomous:

$$\dot{x} = A(t)\,x \qquad , \quad A(t) \in \mathbb{R}^{n\times n}$$

Let $X(t) \in \mathbb{R}^{n\times n}$ denote a fundamental system, i.e.

$$\dot{X} = A(t)\,X \quad \text{and} \quad X(t) = (x_1(t), \dots, x_n(t)),$$

where $x_1(t), \dots, x_n(t)$ are linearly independent solutions.

Consider $\det(X(t)) \mathrel{\hat{=}}$ n-dimensional Volume-element.