Math 525

Differential Equations

Text: J. C. Robinson, Infinite-Dimensional Dynamical Systems, Cambridge Univ. Press, 2001.

1) Introduction

- Poincaré-map
- Floquet-theory
- The Lorenz equations

- Infinite dimensional dynamical systems. PDE's, Reaction-Diffusion equations Navier-Stokes equations
- weak solutions, Sobolev-spaces
- Global Attractors
- Lyapunov-exponents, Lyapunov-multipliers
- Fractal- and Hausdorff-dimension, finite dimensional attractors
- The squeezing property and inertial manifolds.

(1.1) ODE's $\quad \dot{x} = f(x), \quad x \in \mathbb{R}^n$

$f: \mathbb{R}^n \to \mathbb{R}^n$ Lipschitz continuous.

Initial condition $x(0) = x_0$, solution $x(t; x_0)$

Flow: $T(t) := x(t; \cdot)$, $x(t, x_0) = T(t)x_0$

Dynamical system: $(\mathbb{R}^n, T(t))$

Steady states: $f(\bar{x}) = 0$

stability of the steady states from the eigenvalues of the linearization

$$A = Df(\bar{x}) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1}(\bar{x}) & \cdots & \frac{\partial f_n}{\partial x_1}(\bar{x}) \\ \vdots & & \\ \frac{\partial f_n}{\partial x_1}(\bar{x}) & \cdots & \frac{\partial f_n}{\partial x_n}(\bar{x}) \end{pmatrix},$$

eigenvalues: $\mu_1, \dots, \mu_n \in \mathbb{C}$

Theorem 1: $\mathrm{Re}\,\mu_j < 0$ for all $j = 1, \dots, n$
$\Longrightarrow$ $\bar{x}$ is asymptotically stable

Theorem 2: $\mathrm{Re}\,\mu_i < 0 \quad i = 1, \dots, k$

$\mathrm{Re}\,\mu_i = 0 \quad i = k+1, \dots, l$

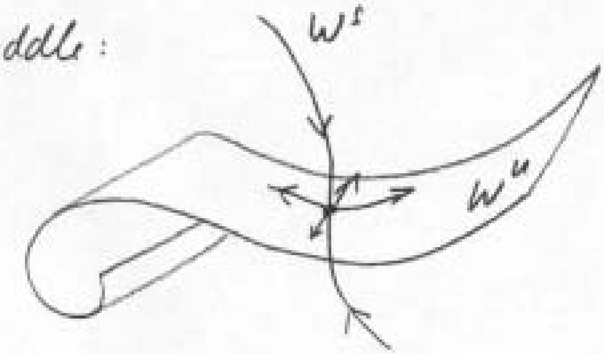
$\mathrm{Re}\,\mu_j > 0 \quad i = l+1, \dots, n$

$\Rightarrow$ there exist k-dim stable manifold $W^s(\bar{x})$

$l-k$ dim. center manifold $W^c(\bar{x})$

$n-l$ dim. unstable manifold $W^u(\bar{x})$

e.g. 3-D saddle:



<u>In 2-D</u>: the <u>zoo</u>: saddle, node, spiral, etc.

$$\mu_{1/2} = \frac{trA}{2} \pm \frac{1}{2}\sqrt{(trA)^2 - 4\det A}$$