

**University of Alberta**  
**Department of Mathematical & Statistical Sciences**

Math 525 Q1  
Ordinary Differential Equations IIB  
Winter 2015

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Office Hours: MWF 10:00-10:45, right after class, or by appointment.  
Lecture Room & Time: MWF 9:00-9:55, CAB 457

Course Web Page: [www.math.ualberta.ca/~thillen/Math525.html](http://www.math.ualberta.ca/~thillen/Math525.html)

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**Course Description:**

**Asymptotics; boundary value problems;** Poincare-Bendixson theory. Additional material will be chosen from among the following topics at the option of the instructor: separation; dichotomies; comparison and oscillation theory; bifurcation theory; nonautonomous systems; **dynamical systems;** functional differential equations; contingent equations; **differential equations in Banach spaces.** Prerequisite: MATH 524 or equivalent.

**Course Objectives and Expected Learning Outcomes:**

In this course we will study asymptotics of ordinary differential equations and boundary value problems. The Poincare-Bendixson theory has been covered in Math 524. We cover the theory of dynamical systems and differential equations in Banach spaces. The concepts of stability and bifurcations can be generalized from ODEs to PDEs. We will systematically derive a theory of finite dimensional compact global attractors, and we will investigate two important examples in detail: the Navier-Stokes equations and reaction-diffusion equations.

**Grade Evaluation:**

There are six assignments of equal weight. I will assign 20 points for each assignment. There is no written exam. An overall course mark of 65% or more guarantees a graduate passing grade of at least C+. An overall course mark of 90% or more guarantees a grade of at least A. Grades are unofficial until approved by the Department and/or Faculty offering the course.

**The due dates are as follows:**

- Assignment 1 (Jan 16, 2015)
- Assignment 2 (Jan 30, 2015)
- Assignment 3 (Wednesday, Feb 25, 2015)
- Assignment 4 (Mar 13, 2015)
- Assignment 5 (Mar 27, 2015)
- Assignment 6 (Apr 10, 2015)

### **Missed Term Work:**

Assignments are due at 9:05 AM in class.

**Late assignments will not be accepted, a late submission results in a grade of 0 for that assignment.**

A student who cannot hand in an assignment due to incapacitating illness, severe domestic affliction or other compelling reasons can apply for an excused absence.

- a) To apply for an excused absence where the cause is incapacitating mental and/or physical illness and most other cases including severe domestic affliction, a student must inform the instructor within two working days following the scheduled date of the term work or term exam missed, or as soon as the student is able, having regard to the circumstances underlying the absence.
- b) For an excused absence where the cause is religious belief, a student must contact the instructor(s) within two weeks of the start of Fall or Winter classes to request accommodation for the term (including the final exam, where relevant). Instructors may request adequate documentation to substantiate the student request.
- c) All other accommodation requests covered by the Duty to Accommodate Procedure should be discussed with the instructor as soon as the student is able, having regard to the underlying circumstance.

An excused absence is a privilege and not a right; there is no guarantee that an absence will be excused. Misrepresentation of Facts to gain an excused absence is a serious breach of the *Code of Student Behaviour*.

### **Tentative Lecture Schedule & Assignments:**

Date	Specials	Contents (tentatively)
Jan 5		1 Introduction (1.1), (1.2) discrete and continuous
7		(1.3) discrete and continuous, (1.4) Wronskian
9		(2.1) Banach spaces (2.2) Mollifiers
12		(2.3) Integral estimates. (2.4) Hilbert spaces
14		(2.5) Linear operators
16	<b>Assignment 1 due</b>	(2.6) Dual spaces
19		(2.6) cont.
21		(2.7) Sobolev spaces, Rellich-Kondrachov
23		(2.8) Banach space valued functions
26		(2.8) cont.
28		3 Reaction-diffusion equations (3.1) Modelling (3.2) Basic assumptions
30	<b>Assignment 2 due</b>	(3.3) Galerkin approximation
Feb 2	No class	
4	No class	
6	No class	
9		(3.3) cont.
11		(3.4) Strong solutions
13		4 Navier Stokes equations (4.1) Intro

16	Reading week, no class	
18	Reading week, no class	
20	Reading week, no class	
23		(4.2) The Stokes operator
25	<b>Assignment 3 due</b>	(4.3) Weak formulation of NS eq.
27	Guest lecture	TBA
Mar 2		(4.4) Weak solutions
4		(4.5) Unique weak solution in 2-D, (4.6)
6		5 Global Attractors (5.1) Attractors
9		(5.1) cont., (5.2) Structure of the attractor
11		(5.3) Shadowing, (5.4) Continuous dependence
13	<b>Assignment 4 due</b>	6 Global attractor for RD eq. (6.1) Absorbing sets
16		(6.1) cont., (6.2) Injectivity
18		(6.3) A Lyapunov function
20		(6.4) The Chaffee Infante eq.
23		7 Global attractor for NS eq. (7.1), (7.2) Injectivity
25		(7.2) cont., 8 Finite dimensional attractors, (8.1) Hausdorff and Fractal dimensions
27	<b>Assignment 5 due</b>	(8.1) cont.
30		(8.1) cont. (8.2) Evolution of volumes
Apr 1		(8.2) cont.
3	Good Friday, no class	
6	Easter Monday, no class	
8		(8.2) cont.
10	<b>Assignment 6 due</b>	(8.3) RD eqs., (8.4) NS eqs.

### **Required Textbook:**

J.C. Robinson. *Infinite-Dimensional Dynamical Systems*. Cambridge University Press, 2001.

### **Recommended or Optional Textbooks**

- R. Temam. *Infinite-Dimensional Dynamical Systems in Mechanics and Physics*. Springer, 1988
- O.A. Ladyzhenskaya, *Attractors for Semigroups and Evolution Equations*, Cambridge 1991.
- M.W. Hirsh, S. Smale. *Differential Equations, Dynamical Systems, and Linear Algebra*. Academic Press, 1974.
- L. Perko. *Differential Equations and Dynamical Systems*. Springer, 3rd ed., 2001
- A.V. Babin, M.I. Vishik, *Attractors of Evolution Equations*. North-Holland, 1992

## **STUDENT RESPONSIBILITIES**

### **Academic Integrity:**

The University of Alberta is committed to the highest standards of academic integrity and honesty. Students are expected to be familiar with these standards regarding academic honesty and to uphold the policies of the University in this respect. Students are particularly urged to familiarize themselves with the provisions of the *Code of Student Behaviour* (online at

www.governance.ualberta.ca) and avoid any behaviour which could potentially result in suspicions of cheating, plagiarism, misrepresentation of facts and/or participation in an offence. Academic dishonesty is a serious offence and can result in suspension or expulsion from the University.

All forms of dishonesty are unacceptable at the University. Any offense will be reported to the Associate Dean of Science who will determine the disciplinary action to be taken. Cheating, plagiarism and misrepresentation of facts are serious offenses. Anyone who engages in these practices will receive at minimum a grade of zero for the exam or paper in question and no opportunity will be given to replace the grade or redistribute the weights. As well, in the Faculty of Science the sanction for **cheating** on any examination will include **a disciplinary failing grade (NO EXCEPTIONS)** and senior students should expect a period of suspension or expulsion from the University of Alberta.

### **Collaboration on Assignments:**

Every term there are several students who receive academic penalties for copying assignments. Here are some tips to avoid copying on assignments:

- (1) Do not write down something that you cannot explain to your TA or instructor.
- (2) When you are helping other students, avoid showing them your work directly. Instead, explain your solution verbally. Students whose work is copied also receive academic sanctions.
- (3) If you find yourself reading another student's solution, do not write anything down. Once you understand how to solve the problem, remove the other person's work from your sight and then write up the solution to the question yourself. Looking back and forth between someone else's paper and your own paper is almost certainly copying and will result in academic sanctions for both you and your fellow student.
- (4) If the instructor or TA writes down part of a solution in order to help explain it to you or the class, you cannot copy it and hand it in for credit. Treat it the same way you would treat another student's work with respect to copying, that is, remove the explanation from your sight and then write up the solution yourself.
- (5) There is often more than one way to solve a problem. Choose the method that makes the most sense to you rather than the method that other students happen to use. If none of the ideas in your solution are your own, there is a good chance it will be flagged as copying.

**Cell Phones:** Cell phones are to be turned off during lectures, labs and seminars.

### **Recording and/or Distribution of Course Materials:**

Audio or video recording, digital or otherwise, of lectures, labs, seminars or any other teaching environment by students is allowed only with the prior written consent of the instructor or as a part of an approved accommodation plan. Student or instructor content, digital or otherwise, created and/or used within the context of the course is to be used solely for personal study, and is not to be used or distributed for any other purpose without prior written consent from the content author(s).

**Students Eligible for Accessibility-Related Accommodations (students registered with Specialized Support & Disability – SSDS):** Eligible students have both rights and responsibilities

with regard to accessibility-related accommodations. Consequently, scheduling exam accommodations in accordance with SSDS deadlines and procedures is essential. Please note adherence to procedures and deadlines is required for U of A to provide accommodations. Contact SSDS ([www.ssd.s.ualberta.ca](http://www.ssd.s.ualberta.ca)) for further information.

**Student Success Centre:** Students who require additional help in developing strategies for better time management, study skills, or examination skills should contact the Student Success Centre (2-300 Students' Union Building).

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Policy about course outlines can be found in section 23.4(2) of the University Calendar.

**Disclaimer:** Any typographical errors in this Course Outline are subject to change and will be announced in class.

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