Homework 2

(due at 11:00 am on May 3, 2010)

Problem 1. Show that for the gas with Maxwellian distribution the average relative velocity of molecules is of the same order as the average thermal velocity and the speed of sound.

Problem 2. Derive Bernoulli's equation and its analogue for potential unsteady flow. Discuss the limits of applicability. Use §§5 and 9 of ref. 3.

Problem 3. Under the influence of surface tension σ, a liquid rises to a height H in a glass tube of diameter D. How does H depend on the parameters of the problem? Use dimensional analysis to reveal this dependence.

Problem 4. Construct the solution for inviscid and viscous plane stagnation-point flow, cf. figure 1.

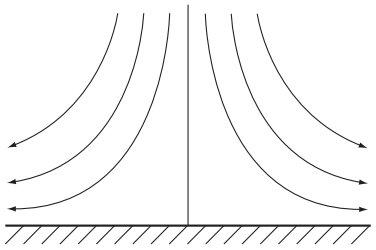

Figure 1: 2D flow near a stagnation point.

Problem 5. Using affine transforms, find the solution for a submerged jet (liquid ejected from a pipe in the space filled with the same liquid, cf. figure 2) in a half-plane $x > 0$, $-\infty < y < +\infty$:

$$\begin{aligned} u\,u_x + v\,u_y &= u_{yy}, \\ u_x + v_y &= 0, \\ |u| &\to 0,\ y \to \pm\infty. \end{aligned}$$

Here (u, v) is the velocity field with (x, y)-components, respectively.

Problem 6. Using affine transforms, find the solution for an axisymmetric drop spreading

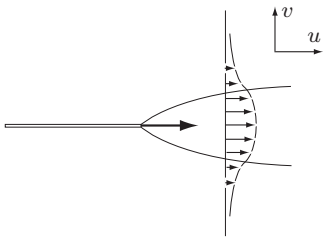

Figure 2: Submerged jet.

on a flat surface, cf. figure 3, described by the following equation

$$\frac{\partial h}{\partial t} = \frac{2}{3r}\frac{\partial}{\partial r}\left(rh^3\frac{\partial h}{\partial r}\right),$$

with the boundary condition $h = 0$ at $r = \infty$ and a mass conservation condition, i.e. mass of the drop should be constant. Make use of a physically relevant conservation law. Determine short and long-time behavior of the solution.

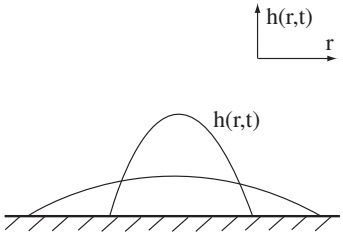

Figure 3: Spreading drop.

Problem 7. Construct self-similar solution for turbulent mixing layer formed by the uniform flow leaving the rectangular body, cf. figure 4.

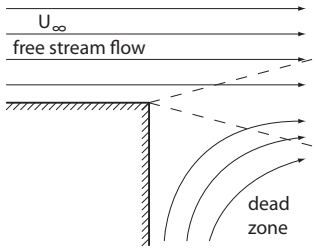

Figure 4: Turbulent mixing layer.