Homework 1

(due at 11:00 pm on April 19, 2010)

Problem 1. Prove the following vector identities

$$(a) \qquad \nabla \times (\boldsymbol{u} \times \boldsymbol{v}) = (\boldsymbol{v} \cdot \nabla)\boldsymbol{u} - (\boldsymbol{u} \cdot \nabla)\boldsymbol{v} + \boldsymbol{u}(\nabla \cdot \boldsymbol{v}) - \boldsymbol{v}(\nabla \cdot \boldsymbol{u}),$$

$$(b) \qquad \nabla(\boldsymbol{u} \cdot \boldsymbol{v}) = (\boldsymbol{u} \cdot \nabla)\boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{u} + \boldsymbol{u} \times (\nabla \times \boldsymbol{v}) + \boldsymbol{v} \times (\nabla \times \boldsymbol{u}).$$

Problem 2. Deduce the vorticity form of the NSEs for incompressible fluid in 2D and 3D.

Problem 3. Derive the evolution equation for the kinetic energy of incompressible viscous fluid in (a) a volume bounded by a solid boundary, and (b) an infinite channel.

Problem 4. Estimate density variation in a compressible isentropic flow.

Problem 5. Using the ideas from kinetic theory of gases, estimate dynamic viscosity of the ideal gas in terms of (1) mean free path, number of molecules per unit volume, mass of a molecule, mean velocity of molecules, and (2) effective collision area, mass of a molecule, Boltzmann constant, and temperature.

Problem 6*. Introduce a stream-function for steady compressible flow. *Hint*: find a coordinate transformation and, possibly, new velocity field to reduce the continuity condition for compressible fluid to the incompressible one.

Problem 7*. Consider fluid occupying a half-space. At its free surface there are a constant pressure p_0 and a constant shear stress T applied. Determine how much time it takes for the velocity at depth z to take its half-value of the velocity at the free surface. Calculate the value for water and the depth of 100 m. Give your physical interpretation of the result.