Homework 2

(due at 3:30 pm on October 26, 2010)

Problem 1. Using the ideas from kinetic theory of gases, estimate dynamic viscosity of the ideal gas in terms of (1) mean free path, number of molecules per unit volume, mass of a molecule, mean velocity of molecules, and (2) effective collision area, mass of a molecule, Boltzmann constant, and temperature.

Problem 2. Derive Bernoulli's equation and its analogue for potential unsteady flow. Discuss the limits of applicability. Use §§5 and 9 of ref. 3.

Problem 3. Under the influence of surface tension σ, a liquid rises to a height H in a glass tube of diameter D. How does H depend on the parameters of the problem? Use dimensional analysis to reveal this dependence.

Problem 4. Construct the solution for inviscid and viscous plane stagnation-point flow, cf. figure 1.

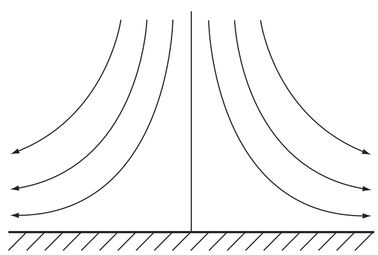

Figure 1: 2D flow near a stagnation point.

Problem 5. Using affine transforms, find the solution for a submerged jet (liquid ejected from a pipe in the space filled with the same liquid, cf. figure 2) in a half-plane $x > 0$, $-\infty < y < +\infty$:

$$u\, u_x + v\, u_y = u_{yy},$$

$$u_x + v_y = 0,$$

$$|u| \to 0, \ \ y \to \pm\infty.$$

Here (u, v) is the velocity field with (x, y)-components, respectively.

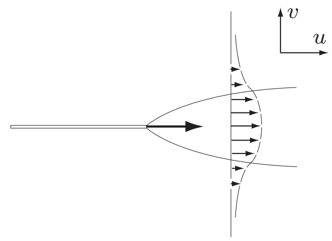

Figure 2: Submerged jet.

Problem 6. Using affine transforms, find the solution for an axisymmetric drop spreading on a flat surface, cf. figure 3, described by the following equation

$$\frac{\partial h}{\partial t} = \frac{2}{3r}\frac{\partial}{\partial r}\left(rh^3\frac{\partial h}{\partial r}\right),$$

with the boundary condition $h = 0$ at $r = \infty$ and a mass conservation condition, i.e. mass of the drop should be constant. Make use of a physically relevant conservation law. Determine short and long-time behavior of the solution.

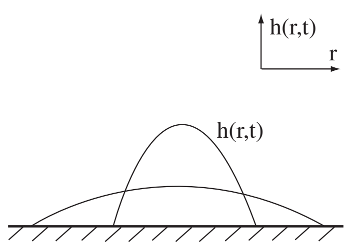

Figure 3: Spreading drop.