Homework 1

(due at 3:30 pm on October 12, 2010)

Problem 1. Prove the following vector identities

$$(a) \qquad (\boldsymbol{a}\times\boldsymbol{b})\cdot\boldsymbol{c}=\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c}),$$

$$(b) \qquad \boldsymbol{t}\times(\boldsymbol{u}\times\boldsymbol{v})=\boldsymbol{u}(\boldsymbol{t}\cdot\boldsymbol{v})-\boldsymbol{v}(\boldsymbol{t}\cdot\boldsymbol{u}),$$

$$(c) \qquad \nabla\times(\boldsymbol{u}\times\boldsymbol{v})=(\boldsymbol{v}\cdot\nabla)\boldsymbol{u}-(\boldsymbol{u}\cdot\nabla)\boldsymbol{v}+\boldsymbol{u}(\nabla\cdot\boldsymbol{v})-\boldsymbol{v}(\nabla\cdot\boldsymbol{u}),$$

$$(d) \qquad \nabla(\boldsymbol{u}\cdot\boldsymbol{v})=(\boldsymbol{u}\cdot\nabla)\boldsymbol{v}+(\boldsymbol{v}\cdot\nabla)\boldsymbol{u}+\boldsymbol{u}\times(\nabla\times\boldsymbol{v})+\boldsymbol{v}\times(\nabla\times\boldsymbol{u}).$$

Problem 2. Consider the vector $\boldsymbol{w}=\boldsymbol{n}\times(\boldsymbol{v}\times\boldsymbol{n})$, where $\boldsymbol{v}$ is arbitrary and $\boldsymbol{n}$ is a unit vector. In which direction does $\boldsymbol{w}$ point, and what is its magnitude?

Problem 3. Deduce the vorticity form of the NSEs for incompressible fluid in 2D and 3D.

Problem 4. Derive the evolution equation for the kinetic energy of incompressible viscous fluid in (a) a volume bounded by a solid boundary, and (b) an infinite channel.

Problem 5. Estimate density variation in a compressible isentropic flow.

Problem 6. If the entropy s is considered as the dependent variable, what are the proper definitions for T, p, and μ (the chemical potential)?

Problem 7. Find the isothermal compressibility coefficient α and the bulk expansion coefficient β for a perfect gas.

Problem 8. Reduce the continuity equation for 2D unsteady compressible flow to the one for 2D incompressible flow using the transformation:

$$\tau=t,\ \xi=x,\ \eta=\int_0^y \rho(t,x,y)\,\mathrm{d}y, \qquad (1)$$

and redefining the y-component of velocity appropriately. *Hint*: use the conection operator in your considerations.

Problem 9. Consider fluid occupying a half-space. At its free surface there are a constant pressure p_0 and a constant shear stress T applied. Determine how much time it takes for the velocity at depth z to take its half-value of the velocity at the free surface. Calculate the value for water and the depth of 100 m. Give your physical interpretation of the result.