Series Convergence Flowchart

$$\sum_n a_n$$

does $a_n \to 0$? — - - → **Diverges** by Divergence Test

Is $a_n > 0$? — - - → Is it alternating in sign and $|a_n|$ decreasing? → **Converges** by Alternating Series Test

Try to show absolute convergence i.e $\sum |a_n|$ converges

Are there any easy comparisons? → **Try Comparison Test:**
if $a_n < b_n$ and $\sum b_n$ converges then $\sum a_n$ converges
if $b_n < a_n$ and $\sum b_n$ diverges then $\sum a_n$ diverges

Does it feel like a_n 'looks like' some b_n? → **Try Limit Comparison Test:**
$\lim \dfrac{a_n}{b_n} = c$ then:
if $0 < c < \infty$ then $\sum a_n$ converges $\Leftrightarrow \sum b_n$ converges
if $c = 0$ and $\sum b_n$ converges then $\sum a_n$ converges.
if $c = \infty$ and $\sum b_n$ diverges then $\sum a_n$ diverges.

Is $\sqrt[n]{|a_n|}$ easy to analyze? → **Try Root Test:**
$\lim \sqrt[n]{|a_n|} = c$
if $0 \le c < 1$ then $\sum a_n$ converges
if $c > 1$ then $\sum a_n$ diverges
if $c = 1$ then test is inconclusive

Is $\left|\dfrac{a_{n+1}}{a_n}\right|$ easy to analyze? → **Try Ratio Test:**
$\lim \left|\dfrac{a_{n+1}}{a_n}\right| = c$
if $0 \le c < 1$ then $\sum a_n$ converges
if $c > 1$ then $\sum a_n$ diverges
if $c = 1$ then test is inconclusive

Is there a positive, decreasing $f(x)$ where $f(n) = a_n$ and $\int f(x)\,dx$ isn't so bad? → **Try Integral Test:**
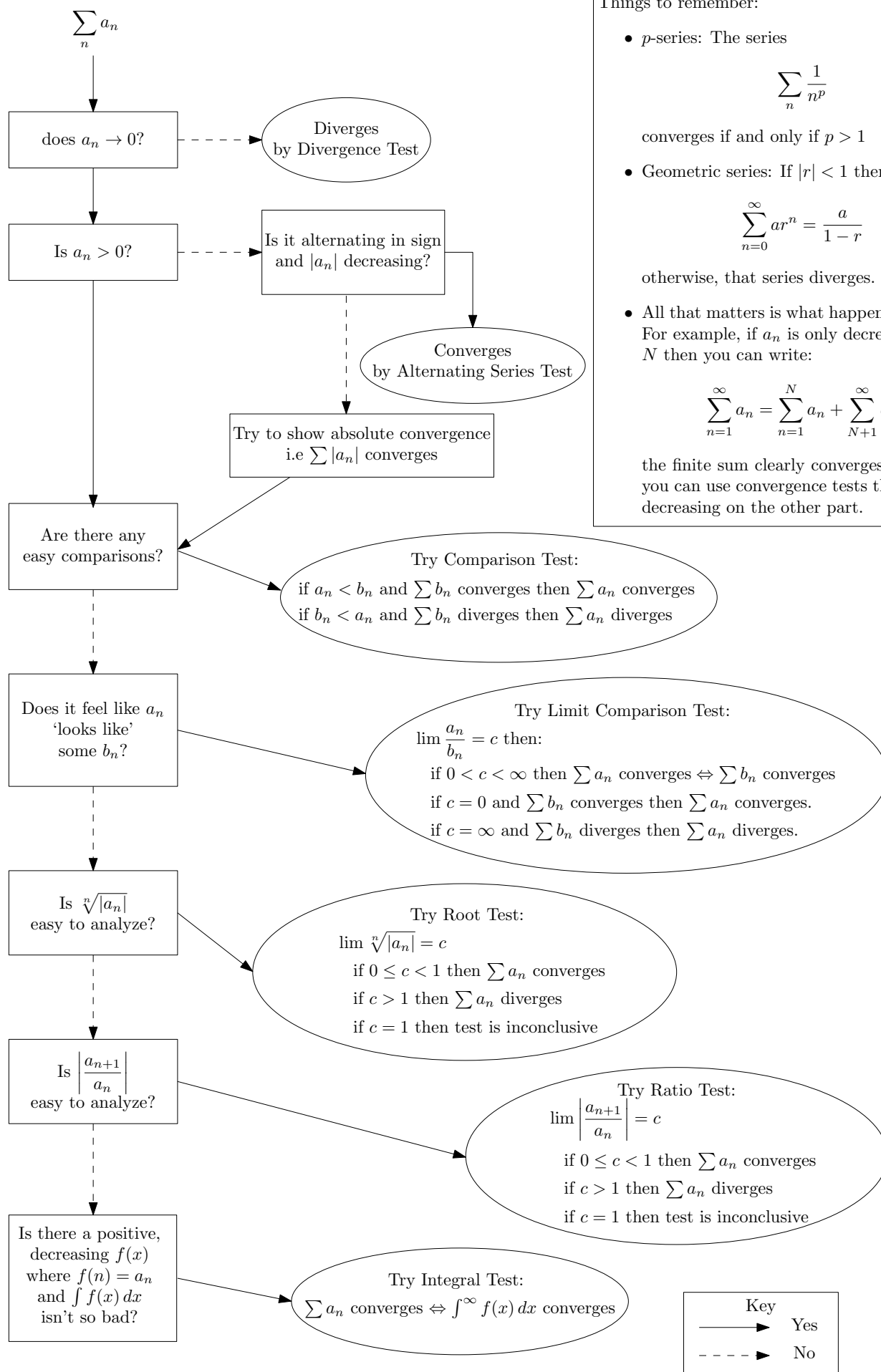$\sum a_n$ converges $\Leftrightarrow \int^{\infty} f(x)\,dx$ converges

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Things to remember:

- p-series: The series
$$\sum_n \frac{1}{n^p}$$
converges if and only if $p > 1$

- Geometric series: If $|r| < 1$ then
$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$
otherwise, that series diverges.

- All that matters is what happens on a tail. For example, if a_n is only decreasing after N then you can write:
$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{N} a_n + \sum_{N+1}^{\infty} a_n$$
the finite sum clearly converges, and then you can use convergence tests that require decreasing on the other part.

Key
— → Yes
- - - → No