Math 225 (Q1) Homework Assignment 8.

1. Show that the second axiom in the definition of a vector space V (that is, the commutative law) follows from the other 7 axioms. Hint: Consider $(1+1)(\underline{u}+\underline{v})$. Recall: The 8 vector space axioms are: for all $\underline{u}$, $\underline{v}$, $\underline{w}$ in V and for all $c, d \in \mathbf{R}$, we have

(A1) $(\underline{u}+\underline{v})+\underline{w}=\underline{u}+(\underline{v}+\underline{w})$

(A2) $\underline{u}+\underline{v}=\underline{v}+\underline{u}$

(A3) there exists $\underline{0} \in V$, independent of $\underline{u}$, such that $\underline{u}+\underline{0}=\underline{0}+\underline{u}=\underline{u}$

(A4) there exists $(-\underline{u}) \in V$ such that $\underline{u}+(-\underline{u})=(-\underline{u})+\underline{u}=\underline{0}$

(A5) $1\underline{u}=\underline{u}$

(A6) $c(d\underline{u})=(cd)\underline{u}$

(A7) $(c+d)\underline{u}=(c\underline{u})+(d\underline{u})$

(A8) $c(\underline{u}+\underline{v})=(c\underline{u})+(c\underline{v})$

2. Let $\underline{b_1}=\begin{pmatrix}1\\-1\\-3\end{pmatrix}$, $\underline{b_2}=\begin{pmatrix}-3\\4\\9\end{pmatrix}$, $\underline{b_3}=\begin{pmatrix}2\\-2\\4\end{pmatrix}$, and $\underline{x}=\begin{pmatrix}8\\-9\\6\end{pmatrix}$. Find the coordinate vector $[\underline{x}]_{\mathcal{B}}$ of $\underline{x}$ relative to the basis $\mathcal{B}=\{\underline{b_1},\underline{b_2},\underline{b_3}\}$.

3. Let S be a finite set of vectors in a vector space V with the property that every $\underline{x} \in V$ has a unique representation as a linear combination of elements of S. Show that S is a basis of V.

4. Let $\mathbf{P}^3$ denote the vector space of polynomials in t of degree less than or equal to 3. Show that $\{1, 2t, -2+4t^2, -12t+8t^3\}$ is a basis of $\mathbf{P}^3$. What is the dimension of $\mathbf{P}^3$?

5. Let A be a $m \times n$ matrix. Show that

(a) If P is a $m \times m$ invertible matrix, then $\operatorname{rank}(PA)=\operatorname{rank}(A)$. Hint: Recall Question 5 in Assignment 7.

(b) If Q is a $n \times n$ invertible matrix, then $\operatorname{rank}(AQ)=\operatorname{rank}(A)$.